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LOGARITHMS

AND

PLANE TRIGONOMETRY.

CHAPTER I.

DEFINITIONS AND EXPLANATIONS OF LOGARITHMS.

IF we take any series of numbers in arithmetical progression, commencing with 0, as 0, 1, 2, 3, &c., and another series in geometrical progression, commencing with 1, as 1, 2, 4, 8, &c.; and if we then place the two series together, thus—

0,	1,	2,	3,	4,	5,	6,	&c.
1,	2,	4,	8,	16,	32,	64,	&c.

then the numbers in the arithmetical series are said to be the *Logarithms* of the corresponding numbers in the geometrical series—the latter being called the *Natural Numbers.* Thus, taking the two preceding series, the logarithm of 64 is 6; the logarithm of 32 is 5; while the natural number corresponding to the logarithm 6 is 64; and so on.

In the specimen just exhibited we have taken two ascending progressions; but they might, equally well, have been two descending progressions, or the one descending and the other ascending. Logarithms, however, as now* used in practice, are limited to the case of two progressions, either both ascending or both descending;—the former giving the logarithms of integers,—the latter, of fractional numbers.

---

* It was not so in the table of logarithms first given to the world by their inventor.

In the same specimen, also, we have taken an arithmetical progression whose common difference is 1, and a geometrical progression having 2 for the common ratio of its terms; but progressions having any other common difference and ratio would have answered equally well. Thus, we may take the two following:—

0,	2,	4,	6,	8,	10,	&c.
1,	3,	9,	27,	81,	243,	&c.

Here, also, the numbers in the upper line are the logarithms of those in the lower. In the latter case, however, we have what is called a different *System* of Logarithms from that in the former. In the former system, 6 is the logarithm of 64; in the latter, 6 is the logarithm of 27.

There may thus be as many different systems as we please; but for practical use, it is necessary to select and adhere to one. That usually employed now, called *Briggs's System*, is the following:—

Logarithms...0,	1,	2,	3,	4,	&c.
Numbers,.....1,	10,	100,	1000,	10000,	&c.

in which a common difference of 1 in the arithmetical series corresponds to the common ratio 10 in the geometrical.

The common ratio in the geometrical series corresponding to the common difference of 1 in the arithmetical series, is called the *Basis* of the system. Thus, the basis of Briggs's system is 10; the basis in the specimen first exhibited is 2; and the basis in the second is $\sqrt{3}$.

In all the above instances it will be observed that the series of natural numbers is not complete. Thus, in the first example, we want the numbers 3, 5, 6, 7, 9, 10, &c., and their logarithms. In the last, we have no natural numbers between 1 and 10, between 10 and 100, &c., and consequently no logarithms corresponding to such numbers. To supply the logarithms of the intermediate numbers, so as to obtain a complete table, requires skilful applications of the highest principles of arithmetic, and immense labour. It is not the province of this work to explain the methods employed for that purpose; and the labour is now no longer necessary, since the calculation has been repeatedly made already, and the whole series of natural numbers, with their corresponding logarithms, have been arranged in tables ready for use. They have also been laid down on straight rules, called *Gunter's Scales* from their inventor;

on *Sliding Rules* of various kinds; and on *Circular Scales*, and *Circular Sliding Rules.*

It now remains to explain what the uses of logarithms are.

The operations of multiplication, division, and extraction of roots, by common arithmetic, become tedious—and combinations of these still more so—when the numbers are represented by many figures. The object of logarithms is to facilitate such operations. They were first employed systematically for that purpose by Baron Napier of Merchiston, who, after trying other ingenious devices, at last happily discovered this useful auxiliary to the arithmetician.

Logarithms have also other uses, which were not at first contemplated by their inventor; but these belong to the higher branches of Mathematics, and may be found explained in works on the Integral Calculus.

To show, in a simple manner, how tables of logarithms may be applied to the purposes of multiplication, division, &c., let us resume the two series with which our illustration commenced,—viz.,

Logarithms...0, 1, 2, 3, 4, 5, 6, &c.
Numbers......1, 2, 4, 8, 16, 32, 64, &c.

Let it now be required to multiply the number 16 by 4: we take their corresponding logarithms standing above them, namely 4 and 2, and *add* them together: their sum is 6, to which we look forward in the line of logarithms: under the 6 we find 64, which is the product we want. If the scholar choose to carry out the two series further, and to try a variety of other numbers, he may find their products in the same manner, without the trouble of multiplying. A little thought will suggest to him the reason; and that reason he will more readily perceive, if he resolve the numbers in the lower line into their factors, writing the two series thus—

0, 1, 2, 3, 4, 5, 6, &c.
1, 2, 2.2, 2.2.2, 2.2.2.2, 2.2.2.2.2, 2.2.2.2.2.2, &c.*

He will then see that each number in the upper line expresses the number of factors in the corresponding term of the lower line. Now, the scholar must be already familiar with a principle commonly employed in multiplication,—that to multiply by any number is the same as to multiply by its several factors successively: thus, to multiply by 96 gives the same result as to multiply by 12 and again by 8. So, when we are required to multiply 16 by 4, it is the same as to multiply by 2 and by 2; thus 16 × 4 = 16 × 2.2. But 16 is = 2.2.2.2. Therefore 16 × 4 is = 2.2.2.2 × 2.2, or = 2.2.2.2.2.2. That is, we have the factor 2 as many times in the product as in the multiplier and multiplicand together; or the logarithm of the product will be equal to the sum of the logarithms of the multiplicand and multiplier.

---

* The point between the figures is used for the sign of multiplication, the same as ×. Thus 2.2.2=8.

Again: suppose it were required to divide 64 by 16; take their logarithms, 6 and 4, in the above series; the *difference* of these is two. We look for 2 in the line of logarithms, and under it we find 4, which is the quotient required. The student may, in like manner, try the process with a variety of other examples. The reason is similar to that of the process for multiplication.

It will be seen that, if we had our two series complete, we might find the products of any two numbers by adding their logarithms together; and their quotient by taking the difference of their logarithms.

Again: since the square of any number is the product of that number by itself, such square may be found by *doubling* the logarithm of the number. Thus, to square 8, we take its logarithm, which is 3 in the above system; twice 3 is 6; under 6 we find 64, which is the square required.

Consequently, the square root of any number may be found by *halving* the logarithm of the number. Thus, to extract the square root of 64, we take its logarithm, 6; the half of 6 is 3; under 3 we find 8, for the root.

To show these principles, we have selected one particular system of logarithms; but they might have been shown equally well in any other system.

The preceding are the *first principles* of logarithms: from these the reasons of the more intricate of the operations explained in the following problems are almost self-evident deductions. Such operations are: the multiplication of several factors; involution and evolution, where powers and roots higher than the second are sought; and processes combining multiplication, division, involution, and evolution.

Logarithms are particularly useful in the practical calculations required in Trigonometry, Plane and Spherical, in Land-surveying, in Compound Interest and Annuities, in Astronomy, and in Navigation.

Other definitions and explanations will be given in the next chapter.

CHAPTER II.

ON THE USE OF LOGARITHMS.

PROBLEM I.

To Find, in Table I, the Logarithm of any Integral Number from* 1 *to* 250.

RULE. Look for the number in the column headed N; and adjacent to the number will be found its logarithm with the proper index.

NOTE 1. In this and subsequent problems, it will be understood that the logarithms in the Tables are not the *exact* logarithms of the numbers given, but merely true to the last decimal figure; for all logarithms, except those of the base and its powers, are interminate decimals.

EXERCISES. What are the logarithms of 5, of 56, and of 231; of 7 and of 70; of 1, of 10, and of 100; of 2, of 20, and of 200; of 23 and of 230?

NOTE 2. In resolving the preceding exercises, the scholar must have observed that the logarithms of 7 and of 70 are alike in their decimal part, and differ only in the integer. He may observe the same thing of 1, 10, and 100; of 2, 20, and 200; and of 23 and 230. This property is peculiar to Briggs's system, and is the cause of that system's being preferred to every other for common purposes. The principle on which the property depends is very simple. Since 20, for instance, is ten times 2, the logarithm of 20 will be the logarithm of 10 added to the logarithm of 2, from what has been said in the last chapter. But the logarithm of 10 is 1. Therefore, the logarithm of 20 will be a unit more than that of 2. For the same reason, the logarithm of 200 will be a unit more than that of 20; that of 2000 will be a unit higher still; and so on. For the same reason, also, the logarithm of 230 is a unit more than that of 23; and the logarithm of 74 is a unit more than that of 7·4.

---

* Table II may be used for the same purpose; but, in general, Table I will be more convenient.

Hence, since the logarithm of 1 is 0, and that of 10 is 1, it follows that the logarithm of 100 is 2, that of 1000 is 3, that of 10,000 is 4, &c. Hence, also, the logarithms of all numbers between 1 and 10 will be between 0 and 1, or will be 0 with a decimal fraction; those of numbers between 10 and 100 will be 1 with a decimal; between 100 and 1000, 2 with a decimal; and so on.

Definition. The integral part of the logarithm is called its *Index*.

Note 3. From what has just been said, it follows that the index of all numbers between 1 and 10 will be 0; that of numbers between 10 and 100, 1; between 100 and 1000, 2, &c. Or the index will always be a unit less than the number of integral figures in the natural number. Thus the index of 4 will be 0; that of 64 will be 1; that of 365, 2; that of 4821, 3; that of 56908, 4; and so on. Hence the Rule for the following Problem.

Problem II.

To find the Index of the Logarithm of any given Number, partly integral, partly decimal.

Rule. Count the number of integral figures. That number, diminished by 1, will be the index.

Thus the logarithmic index of the number 7, of 7·5, of 6·28, or of 3·084, is 0; the index of 63, of 26·7, or of 94·108, is 1; that of 865·24, is 2; that of 7654, or of 5836·1, is 3; and so on.

Exercise. What are the logarithmic indices of the natural numbers 5, 76, 3920, 69·4, 706, 521·85, 1·6, 3·007, and 581,296?

NOTE. *When the given number is a decimal fraction without integers*, the index is negative and is written thus, $\bar{2}$. We observe in what place of decimals, in the given number, the first significant figure occurs, and the number of that place, with the negative sign written over it, is the index. Thus the index of ·582 is $\bar{1}$; that of ·058 is $\bar{2}$; that of ·006 is $\bar{3}$; and so on. But, since the negative indices are troublesome to beginners, they will be dispensed with altogether in this volume, and rules given for working every question without them.

PROBLEM IV.

To find, in Table II, the Logarithm of any number consisting of not more than three Figures.

NOTE 1. When we say, in this place, "three figures," we mean independently of ciphers either to the right or to the left. Thus we should include 6340, 73000, and ·00265 under the head of this problem.

RULE. *If the number consists of three figures*, look for it in the left-hand column of the table; and opposite it, in the next column, will be found its logarithm, but without the index. The index must be found by Problem II, and inserted.

Thus the logarithm of 576 is 2·760422; that of 39·4 is 1·595496; that of ·0253 is $\bar{2}$·403121.

If the number consists of less than three figures,* make up three by placing ciphers, if not already present (or by supposing them placed), on the right of the number, such ciphers being always regarded as decimals. Then proceed as before.

Thus the logarithm of 75 is the same as that of 75·0 the logarithm of 8 is the same as of 8·00; and that of ·035 is the same as of ·0350.

NOTE 2. In both cases a decimal point occurring in the given number is taken no notice of till we come to the insertion of the index.

---

* In this case the logarithm may be found also from Table I, by changing the index, if necessary.

NOTE 3. When a blank occurs in the left side of the column of logarithms, its place must be filled up by taking in the two figures next above it in the same column.

EXERCISE 1. What are the logarithms of 563, of 708, and of 550?

2. What of 365, of 206, and of 900? (See Note 3).

3. Find the logarithms of the numbers 24·6, and 3·88. (See Note 2).

4. Find the logarithms of 8,540 and of 73,000. (See Note 1).

5. What are the logarithms of 27, of 3·9, and of 8·0?

PROBLEM V.

To find, from Table II, the Logarithm of any Number represented by four Figures, integral or decimal.

RULE. Look for the three first figures of the number in the first column of the table. Having found these, we have the *line* in which the required logarithm is to be found. Then look for the fourth figure of the given number in the line of digits at the top or at the bottom of the same page. This will show the *column* in which the required logarithm is contained. Let the eye then move along the *line* previously found till it come to the proper *column*, it will then rest on the logarithm sought, or rather on the four last figures of that logarithm. The two other figures are to be taken from the left-hand side of the left-hand column of logarithms, taking the two at the beginning of the same line, unless their place is blank; but, if blank, then the two next above the blank or the two next below it. The two next above are always to be taken unless the first of the four figures previously found has a hyphen* over it, thus $\bar{2}$; in which case we take the two next below. Having written these two figures before the four previously found, prefix the decimal point, and the index, found as before.†

---

* The mark is the same as that for the negative sign over an index; but, from its position, it can never be mistaken for it.

† The teacher will probably find it necessary to explain this rule to the learner, and to see him practise it. The rule *seems* complicated, but becomes easy enough after the first trial.

Thus the logarithm of 3470 is 3·540329; the logarithm of 3478 is 3·541330; that of 3·492 is 0·543074; and that of 0·3468 is $\bar{1}$·540079.

NOTE. Observe Notes 1 and 2 of the last problem.

EXERCISE 1. What are the logarithms of the numbers 4580, 4586, 87·40, 87·42, 3·357, and 933·4?

2. What of 49,210, of 7;836,000, of 380,800, and of 72;000,000?

3. Find the logarithms of 25,860, of 2,586, of 258·6, of 25·86, and of 2·586.

PROBLEM VI.

To find the Logarithm of any Number represented by more than four Figures.

RULE. Regard all the figures after the first four as ciphers, and find the logarithm by Problem V.

If the first of the neglected figures is greater than 5, or if it is 5 followed by other significant figures, the preceding figure should be increased by a unit on dropping those that follow.*

Thus, instead of 54682, we look for 54680; and, instead of 93·6481, we look for 93·65.

EXERCISE. Find the logarithms of the numbers 93,514; 48,627; 78,362; 9,244·8; 3·2764; 832,962; 12,954·8; 300·424; and 168;544,919.

Answers: 4·970858; 4·686904; 1·894094; 3·965907; 0·515344; 5·920645; 4·112270; 2·477700; and 8·226600.

PROBLEM IX.

To find the Natural Number answering to any given Logarithm.

RULE. Observe the two figures first after the index, and look for them in the left margin of the first column of logarithms in Table II. Having found these, look along the same and the succeeding lines* till the eye rest on the other four figures, *if they can be found exactly*. If so, take the three figures of the same line in the column N, and write after these the figure at the head and foot of the column in which the last four figures of the logarithm were found.

---

* This rule is sufficiently accurate for all questions in an elementary course. For those requiring more accurate calculation, the student is referred to Rule II of this Problem in the Author's larger work on the subject.

But *if the last four figures of the logarithm cannot be found exactly in the table*, take the logarithm which comes nearest to it in the table, either the next less or the next greater, and the number answering to this will be near enough for ordinary purposes. It will be true to the fourth figure.

When the figures of the natural number have thus been found, place the decimal point so that the number of integral figures may be one more than the index denotes. Ciphers must be supplied to the right, if necessary, to make up the number.

NOTE. If the index is negative, place the decimal point to the left of the natural number found, along with as many ciphers as may remove the first significant figure to that place of decimals which the index expresses; that is, one cipher fewer than the number denoted by the index.

EXAMPLE. What is the natural number corresponding to the logarithm 2·184030, true to four figures?

Given logarithm,...... 2·184030
Nearest log. on Table, 2·184123

Corresponding to the latter, we have the natural number 152·8.

Answer: 152·8 –.

EXERCISE 1. What are the natural numbers corresponding to the logarithms 5·000000; 3·130334; 2·472318; 1·972434; and 0·725013?

Answers: 100,000; 1350; 296·7; 93·85; and 5·309.

2. What are the natural numbers of which the follow-

---

* Or the latter part of the preceding line, when any of the figures in that line are marked with a hyphen above.

ing are the logarithms, 0·860098 and 3·181272? (See the last foot-note). Answers: 7·246 and 1518.

3. The same with the logarithms 2·971278, 3·892201, 0·752286, and 1·270199?

Answers: 936·0 +, 7802 −, 5·653 +, and 18·63 −.[*]

Problem X.

To find the product of two or more Numbers by means of Logarithms.

Rule. Add together their logarithms, and find the natural number answering to the sum.

Example 1. What is the product of the numbers 7832, 4·526, and 203·8?

Numbers.	Logarithms.
7832	3·893873
4·526	0·655715
203·8	2·309204
Ans. 7;224,000	6·858792.

The answer is here found correct only to four figures, the other places being supplied with ciphers.

Example 2. Multiply 54·023 by 2·5296.

Num. 54·02[†]	Log. 1·732555
2·530	0·403121
Ans. 136·7 −	2·135676.

Note. *If one or more of the factors are decimals without integers*, in all such factors remove the decimal point just so many places to the right as will render the first significant figure an integer. Find, by the rule, the product of the factors thus altered, and then remove the decimal point in the product as many places to the left as the decimal points were previously removed to the right in all the altered factors together.

---

[*] The signs + and — will not be invariably expressed in this volume. Whether or not, it will be understood that the numbers found are very rarely definite numbers, but merely correct to the last figure put down.

[†] See the Rule for Problem VI.

Example 4.[*] Multiply 36·94 by ·06857.

Num. 36·94……Log. 1·567497
6·857…………0·836134

253·3 – ………2·403631
Ans. 2·533 –.

Example 5. Multiply ·65023 by ·028518.

Num. 6·502……Log. 0·813047
2·852…………0·455150

18·54 +………1·268197.
Ans. ·01854 +.

Exercise 1. Multiply 725 by 48 by logarithms, and prove the result by common multiplication. Ans. 34800.

2. Multiply 172·3 by 4·598. Ans. 792·2 +.

3. Find the product of the numbers 3·69, 6·4, and 348·5 by logarithms, and prove it by multiplication. Ans. 8230 +.

4. What is the continued product of the numbers 96, 125, 39, and 88·4, true to four figures, supplying the place of the others with ciphers. Ans. 41,370,000.

5. Multiply 5·2763 by 634·29. (See Example 2). Ans. 3347 –.

6. What is the product of the three factors, 2·8539, 36·912, and 3·4846. Ans. 367·1 +.

7. Multiply 49·12 by ·05835. (See Note 1). Ans. 2·866 +.

8. Multiply ·48513 by ·036959. Ans. ·01793 –.

9. Multiply together 63·42, 5·897, ·4081, ·03976, and ·1648. Ans. 1·000 +.

[*] The numbers of the Problems, Examples, and Exercises are retained as in the Author's "Treatise." The omitted numbers belong to the more advanced course.

PROBLEM XI.

To divide one Number by another, by means of Logarithms.

RULE. From the logarithm of the dividend subtract the logarithm of the divisor, and find the natural number answering to the remainder.

EXAMPLE 1. Divide 3468·3 by 44·689.

Num.	3468.........Log.	3·540079
	44·69..............	1·650210
Ans.	77·60+............	1·889869.

NOTE 1. *When the divisor is greater than the dividend,* move the decimal point, in the latter, as many* places to the right as will render it greater than the divisor; and, when the quotient is found from the dividend thus altered, restore the decimal point in the quotient to its proper place, by moving it as many places to the left as it was previously moved to the right in the dividend.

EXAMPLE 3. Divide 34·264 by 8564·9.

Num.	34260............Log.	4.534787
	8565..................	3·932727
	4·000..................	0·602060.
Ans.*	·004.	

EXAMPLE 4. Divide 34 by 582.

Num.	3400.. ....Log.	3·531479
	582..................	2·764923
	5·842 –...............	0·766556.
Ans.	·05842 –.	

NOTE 2. *When the divisor is a decimal fraction, without integers, and less than the dividend,* remove the decimal point

of the divisor as many places to the right as will render the first figure an integer; and then remove the decimal point of the dividend as far to the right as that of the divisor has been removed. When this is done the quotient will come out correct at once, without requiring any alteration.

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* There is no objection to removing the decimal point as far to the right as we please, provided that it be restored the same number of places in the quotient. The direction given in other books is to remove it, or rather to suppose it removed, *ten* places. A similar remark is applicable to Note 2.

Example 5. Divide ·4828 by ·01958.

Num. 48·28............Log. 1·683767
1·958.................. 0·291813

Ans. 24·66 – .... 1·391954.

Note 3. *But when the dividend is a decimal fraction without integers, and less than the divisor*, remove the decimal point of the divisor (if necessary) as many places to the right as will render the first figure an integer, and that of the dividend as many as will render it greater than the divisor. After both changes, observe how many places the point has been removed in the dividend more than in the divisor, and restore it that number of places in the quotient.

Example 6. Divide ·01958 by ·4828.

Num. 19·58............Log. 1·291813
4·828.................. 0·683767

4·056.................. 0·608046.
Ans. ·04056 –.

Exercise 1. Divide 1728 by 144 by logarithms, and prove the result by common division. Ans. 12.

2. Divide 3784 by 26. Ans. 145·5 +.
3. Divide 48·2799 by 2·58381. Ans. 18·68 +.
4. Divide 263·81 by 92·413. Ans. 2·855 –.
5. Divide 36·49 by 192·54. Ans. ·1896 –.
6. Divide 26 by 3784. (See Note 1.) Ans. ·006,871 +.
7. Divide ·00386 by ·00275. (See Note 2.) Ans. 1·404 –.
8. Divide 9·5384 by ·058,262. Ans. 163·7 +.
9. Divide ·378,44 by ·026,703. Ans. 14·17 +.
10. Divide ·4368 by 2·564. (See Note 3.) Ans. ·1704 –.

11. Divide ·8408 by ·9627. Ans. ·8734 –.
12. Divide ·005,842 by ·2382. Ans. ·02453 –.

Problem XII.

To find the fourth Term of a Proportion by Logarithms.

Rule. Add together the logarithms of the second and third terms, and from the sum subtract the logarithm of the first.

Example 1. Find the fourth term of the following proportion. As 657 : 1533 : : 279 : ——.

As 657...............2·817565

Is to 1533...............3·185542
So is 279...............2·445604

To 651...............2·813581.

Note 1. After a little practice, the student will be able to subtract the first logarithm from the sum of the other two at once, without previously setting down the said sum. But, if he find that he cannot do this easily and correctly, his best way is to put down the second and third terms before the first, leaving a space between for the sum of the former, thus:—

1533..................3·185542
279..................2·445604

5·631146
657..................2·817565

Ans. 651..................2·813581.

Note 2. *If the first term is a decimal fraction without integers*, remove the decimal point as many places to the right as will render the first figure an integer, and make compensation for so doing, by removing the decimal point an equal number of places to the right in the second or third term, or in both together.

Example 3. As ·36 : 1 : : 728 : ——.

As 3·6..............0·556303

Is to 10..............1·000000

So is 728..............2·862131

To 2022 +...........3·305828.

Example 4. As ·000,449 : ·1876 : : ·05357 : ——.

As 4·49.............. 0·652246

Is to 18·76.............. 1·273233

So is 5·357.............. 0·728922

To 22·38 +........... 1·349909.

Note 3. After the operation directed in the preceding note, or when that is not required, *if the second or third term is a decimal fraction without integers*, remove the decimal point as far to the right as will render the first figure an integer, and make compensation for it in the fourth term, when found, by returning the point equally far to the left.

Example 5. As 7·914 : ·02833 : : 6·687 : ——.

As 7·914.............. 0·898396

Is to 2·833.............. 0·452247

So is 6·687.............. 0·825231

To 2·394 −........... 0·379082.

Ans. ·02394 −.

Example 6. As 0·336 : 0·484 : : ·095 : ——.

Here we first proceed by Note 2, changing the first and second terms to 3·36 and 4·84. After that, since the third remains in a fractional form, we make it 9·5, and make compensation by restoring the decimal point two places to the left in the answer.

As 3·36..............0·526339

Is to 4·84...............0·684845
So is 9·50........ ......0·977724

To 13·68+...........1·136230.

Ans. 0·1368+.

Note 4. If the removal of the decimal point, as directed in the last note, was necessary both in the second term and in the third, restore the decimal point in the fourth term, by moving it as many places to the left as will be equal to the number of places it was previously removed to the right in the second and third terms together.

Example 7. As 5·33 : 0·28 :: 0·055 : ——.

As 5·33..............0·726727

Is to 2·80...............0·447158
So is 5·50...............0·740363

To 2·889+...........0·460794.

Ans. ·002,889+.

Note 5. When the three given terms commence with integral figures, or when they have been transformed by the directions in the preceding Notes, so as to commence with integers—*if the logarithm of the first term is greater than the sum of the logarithms of the other two terms*, remove the decimal point to the right, in either the second or the third term, as far as may be necessary to render the sum of the two last-mentioned logarithms greater than the logarithm of the first term. Then, after finding the fourth term, count how many places the decimal point has been removed in the second and third terms together, more than in the first, and restore it that number of places in the fourth term.

Example 8. As 0·977 : 0·146 :: 0·045 : ——.

As 9·77..............0·989895

Is to 1·46..............0·164353
So is 45·0..............1·653213

To 5·342–...........0·727673.

Here we find the decimal point has been removed one place in the first term, and four places in the other two terms together; we must therefore restore it three places in the answer.

Ans. ·005342 –.

Exercises. Compute the fourth term of each of the following proportions, by Logarithms, and prove the first three by common Arithmetic.

1. As 1314 : 3066 :: 558 : 1302.
2. As 321·33 : 14·282 :: 39·419 : 1·752. (See Problem VI, Rule I.)
3. As 35·04 : 7·884 :: 2856 : 642·6.
4. As 76 : 849 :: 3·582 : 40·01 +.
5. As 0·3504 : 78·84 :: 2·856 : 642·6. (See Note 2.)
6. As ·00098 : 0·304 :: ·0235 : 7·290 –.
7. As 4·38 : 0·985 :: 379 : 85·23 +. (See Note 3.)
8. As 0·5072 : 0·4814 :: ·0688 : ·06530 +.
9. As 2·69 : 0·548 :: 0·886 : 0·1805 –. (See Note 4.)
10. As 0·9826 : ·04385 :: ·07848 : ·003502 +.
11. As 1314 : 30·66 :: 5·58 : 0·1302. (See Note 5.)
12. As 32·133 : 1·4282 :: 0·39419 : ·01752.

Problem XIII.

To Square or Cube a given Number by Logarithms.

Rule. Multiply the logarithm of the number by 2 for the square, or by 3 for the cube; and find the natural number answering to the product.

Example 1. What is the cube of 12?

Num. 12.........Log. 1·079181
3

Ans. 1728...............3·237543.

Note. *If the given number is a decimal without integers*, remove the decimal point as many places to the right as shall render the first significant figure an integer; and, when the square or cube is found, restore the decimal point in the result to its proper place, by removing it to the left,—in the case of the square, twice as many places as it was previously removed to the right, and, in the case of the cube, three times as many.

Example 2. What is the square of ·0836?

Num. 8·36.........Log. 0·922206

2

69·89................ 1·844412.

Ans. ·006989 –.

Example 3. What is the cube of ·270695?

Num. 2·707............Log. 0·432488

3

19·84 – ....1·297464.

Ans. ·0198 +.

Exercise 1. Square the numbers 12, 65·4, 49·59, and 4·732. Answers: 144, 4277 +, 2459 +, 22·39 +.

2. Cube the numbers 19, 5·832, and 16·28.
Answers: 6859, 198·4 –, and 4315 –.

3. Square the number 793, and cube the same number, giving the answers correct to four figures each, and supplying the remaining places with ciphers.
Answers: 628,800, and 498;700,000.

4. Square the numbers ·01738, ·00256, and ·26954. (See the Note.)
Answers: ·000,302,1 –, ·000,006,554 –, and ·072,65 +.

5. Cube the numbers ·3462, and ·0987.
Answers: ·041,49 +, and ·000,961,5 +.

Problem XV.

To extract the Square Root or Cube Root of any given Number.

Rule. Divide the logarithm of the given number by 2 or by 3. The natural number answering to the quotient will be the square root or the cube root required.

Example 1. What is the cube root of 378·4?

Num. 378·4............3)2·577951 Log.

Ans. 7·233 –......... 0·859317.

Note. *If the given number is a decimal without integers*, count off the figures in groups of two or three each, according as the square or cube root is required, commencing at the decimal point and separating the groups by commas; then remove the decimal point to the place of the comma which follows the first significant figure. When the root of the number thus altered has been found, restore the decimal point to its proper place by removing it to the left as many places as the number of groups it previously passed over to the right. Thus, observe whether the decimal point in the given number was transferred to the place of the first, second, or third comma, and accordingly replace it one, two, or three figures further to the left.

Example 2. Extract the square root of the fraction ·000,003,567.

·00,00,03,56,7.
Num. 3·567.........2)0·552303 Log.

Root 1·889 –...... 0·276151.
Ans. ·001,889 –.

Exercise 1. Extract the square roots of the numbers 144; 2; 47·32; 1·909; 17;690,000; and 384·59.

Answers: 12; 1·414 +; 6·879 –; 1·382 –; 4,206 –; and 19·61 +.

2. Extract the cube roots of the following: 6,859; 36; 3;497,000; 563·1; and 2·5838.

Answers: 19; 3·302 –; 151·8 –; 8·258 –; and 1·372 +.

3. Find the square roots of ·079,63; ·935,62; ·007,26; ·000,537; and ·000,016,9. (See Note).

Answers: ·2822 –; ·9673 –; ·0852 +; ·023,17 +; and ·004,111 –.

4. Find the cube roots of ·802,88; ·005,831; ·000,076; and ·000,000,532,7.

Answers: ·9294 +; ·1800 –; ·042,36 –; and ·008,106 +.

CHAPTER III.

ON THE APPLICATIONS OF LOGARITHMS.

Logarithms are applicable to all operations involving multiplication, division, involution or evolution; and, in combinations of those operations, they save much labour.

Their grand use is in *Trigonometrical calculations*—a use which will be fully explained in the Second Part of this volume. But they may also be applied to all questions under the Arithmetical rules of *Reduction*, *Simple and Compound Proportion*,* *Interest*, *Exchange*, *Annuities*, *Geometrical Progressions*, &c., and to questions in *Mensuration*. Of all these applications, however, it is for the operations of Compound Interest and Annuities, and for finding the areas of triangles when the three sides are given, that logarithms are particularly advantageous and are often employed.

Examples of some of the applications mentioned, and Exercises in them, will be found in the following pages. In these it will be taken for granted that the scholar is already acquainted with the rules of Arithmetic and of Mensuration, leaving nothing more to be exhibited here than the mode of applying the logarithms, which will be a mere exemplification of the rules given in the preceding Chapter.

There are other purposes, both in science and in the higher branches of Mathematics, to which logarithms are applied. But these belong to a more advanced stage of the student's progress, and are invariably explained in treatises on the particular subjects of their application.

EXAMPLE 1. Reduce 764 old corn gallons to imperial gallons.

	Num.	Log.
	764	2·883093
	268·8	2·429429
	277·3†	2·442950
Ans.	740·6	2·869572.

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* Including, of course, the numerous applications of Proportion to which other names are applied, such as *Profit and Loss*, *Discount*, &c.

† This would be 277·274 correctly.

Example 2. If the rent of 38 acres 3 roods of land is £75, 10s., what should be the rent of 884 acres, at the same rate?

As 38·75..............Log. 1·588272

Is to 884.................... 2·946452
So is 75·5.................... 1·877947

To 1,722 +................. 3·236127.
Ans. £1,722 +.

Example 3. If the rent of a farm of 38 acres 3 roods of land, in England, is £75, 10s., what should be the rent of a farm of equal quality in France, containing 1,677 *ares*, supposing it let on the same terms; an imperial acre containing 4,840 square yards, and an *are* of French measure containing 119·6 square yards, imperial?

As 38·75 × 4,840 : 1,677 × 119·6 :: 75·5.

Num.	Logarithm.	Num.	Logarithm.
38·75	1·588272	1677	3·224533
4840	3·684845	119·6	2·077731
		75·5	1·877947
	5·273117.		
			7·180211
			5·273117
Ans. £80 : 14 : 10 −.		80·741	1·907094.

Example 4. What is the interest of £74, 12s., for 96 days, at 4$\frac{1}{2}$ per cent. per annum?

$$\left.\begin{array}{l}\text{As } 100£ \ : \ 74·6 \\ \text{As } 365 \text{ days} \ : \ 96 \text{ days}\end{array}\right\} :: 4·5£$$

Num.	Logarithm.	Num.	Logarithm.
100	2·000000	746*	2·872739
365	2·562293	96·0	1·982271
		4·5	0·653213
	4·562293		
			5·508223
			4·562293
		8·829 +	0·945930.

£·8829 = 17s., 7$\frac{3}{4}$d. Ans.

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* The decimal point is removed one place to the right and restored one place at the conclusion.

Example 5. The first term of a geometrical progression is 5·45, and the common ratio 1·874; what is the tenth term?

$$\text{Tenth term} = 5{\cdot}45 \times 1{\cdot}874^9.$$

Num. 1·874............Log. 0·272770
9

2·454930
5·45..................0·736397

Ans. 1,554 – ....3·191327.

Example 6. What is the compound interest of £854 for six years, at 4½ per cent. per annum?

$$\text{Amount} = 854 \times 1{\cdot}045^6.$$

Num. 1·045............Log. 0·019116
6

0·114696
Principal = 854 ....2·931458

Amount = 1112 ....3·046154.

Comp. Int. = £258, Ans.

Example 14. The diameter of a circle being 25·84, what is its area?

$$\text{Area} = 25{\cdot}84^2 \times \frac{\Pi}{4}.^*$$

Num. 25·84.........Log. 1·412293
25·84............... 1·412293
7·854 – .... 0·895090

5244 + ... 3·719676.
Ans. 524·4 +.

Example 15. Taking the Sun and Earth as spheres,—the former of 883,000 miles in diameter, and the latter of 7,912, how many times the bulk of the Earth is the Sun?

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* For the uses and values of Π and its parts, see Part II, Ch. III, Pr. VI, &c.

$7912^3 : 883,000^3 :: 1$

Num.	Logarithm.	Num.	Logarithm.
7,912......	3·898286	883,000......	5·945961
	3		3
	11·694858		17·837883
			11·694858

Ans. 1,390,000 times............ 6·143025.

EXERCISE 1. An imperial gallon contains 277·274 cubic inches. Reduce 379 cubic feet to gallons.

Ans. 2362 – gals.

2. If the construction of a railway, $48\frac{1}{2}$ miles, cost £736,800, what will it cost to continue the same railway an additional distance of $15\frac{3}{4}$ miles, at the same rate?

Ans. About £239,300.

3. If 939 men consume 702 quarters of corn in 14 months, how many men can be maintained 5 months on 1,464 quarters?

Ans. 5483 + .

4. An estate of 386 imperial acres was purchased for £19,740. A small piece of land lies contiguous, said to contain $3\frac{1}{2}$ Scotch acres, and of equal quality: the owner offers to let me have this on the same terms as the estate. What will be the price, the Scotch acre being equivalent to 1·261 imperial acre?

Ans. £225, 14s. + .

5. What is the interest of £1,578 for 139 days at $4\frac{1}{4}$ per cent. per annum?

Ans. £25 : 10 : $9\frac{1}{2}$+.

8. What is the amount of £320, 10s., for four years, at 5 per cent. per annum, compound interest?

Ans. £389, 12s., nearly.

9. What is the compound interest of £15, 10s., for nine years, at $3\frac{1}{2}$ per cent. per annum?

Ans. £5 : 12 : 6.

19. The diameter of a circular enclosure is 89 yards: how many acres does it contain?

Ans. 1 ac. 1 ro. 5·7 – po.

20. What is the area of a circle whose circumference is 32·25? Ans. 82·77 –.

21. Find the solid content of a cylinder whose length is 42 inches, and diameter 15 inches. Ans. 4·295 + c. feet.

22. Compute the solid content of a regular triangular pyramid, whose perpendicular height is 6·99, each side of the base being 4·18. Ans. 17·63 –.

23. The fixed axis of a prolate spheroid is 7·74 inches, and the revolving axis 5·88 inches. What is its solid content? Ans. 140·1 + c. inches.

24. If a cannon ball, 3½ inches in diameter, weigh 6 lb., what will be the weight of another ball of the same metal, 6·7 inches in diameter? Ans. 42·09 – lb.

CHAPTER IV.

OF LOGARITHMIC SCALES.

The *Logarithmic Scales*, of various kinds, are merely rules on which the lengths of logarithms, taken from a table, are set off by measurement, marked with the corresponding natural numbers.[*] Thus, the figure 6 is marked at the termination of the length of the logarithm of 6. All the lengths commence at one point, which is marked with a unit, since the logarithm of 1 is 0.

Consequently, logarithms of numbers are added together by uniting their lengths upon the scale; and one is subtracted from another by measuring the length of the former backward from the termination of the latter.

Consequently, also, natural numbers are multiplied together by uniting the lengths of their logarithms on the scale, and observing the marked number to which their united length reaches; and one number is divided by another, by measuring the length corresponding to the latter backward from the termination of the length corresponding to the former.

Sliding Rules consist of two such logarithmic scales sliding along each other, with their graduated edges in close contiguity.

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[*] Except the Trigonometrical scales of sines, tangents, &c., which are not marked with the natural numbers, but with the corresponding degrees.

In using the single logarithmic scale, one of the two given lengths requires to be measured with the compasses; but the sliding rule renders this unnecessary, since the one length is added to the other or subtracted from it by mere position,—that is, by bringing the commencement of the former to the termination of the latter in the one case, and, in the other, by placing the two terminations together.

For scales, Briggs's system of logarithms alone is used, in order that the length from 10 to 100, with its divisions, may be an exact counterpart of that from 1 to 10; since, in that system, as before explained, the logarithm of any number is the same as the logarithm of ten times that number, except that the index differs by a unit.

Logarithmic scales in their usual and simplest form, are called, from their inventor, *Gunter's Scales.*

PROBLEM I.

To find the Product of two Numbers by Gunter's Scale.

RULE. Extend the compasses from 1 to the smaller of the two factors on the line of Numbers. Keep the distance unaltered in the compasses, and it will extend forward from the other factor to the number indicating the product.*

EXAMPLE 1. Multiply 9 by 6.

Operation. Place the compasses on the scale with the left foot on 1 and the right foot on 6. Remove the compasses, without altering the distance, and place the left foot on 9; the right foot will reach to 54, which is the Answer.

NOTE. We cannot measure a number to greater nicety than three figures on the best scales, and scarcely even so far if the first figure is large. If, therefore, the given numbers consist of more than three figures, we must take the third to the nearest figure, whether more or less, and omit the subsequent figures. Thus, for 7·942 we take 7·94, and 34291 we take 34300. In the results we must also be satisfied with three figures, filling up the places of the remainder with ciphers when necessary.

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* The numbers on the first half of the scale must be regarded as units, and those on the second half as tens. When the first figures of the given numbers are not units (or tens in division) the operation requires particular rules, which will be found in the Author's "Treatise on Logarithms and Plane Trigonometry."

EXAMPLE 2. Multiply 3·811 by 7·428.

Operation. 3·81 × 7·43 = 28·3. Ans. 28·3.

COR. The product of two numbers is found *by the Sliding Rule* thus:—Calling one of the two scales A and the other B, place 1 of scale B adjacent to one of the factors on scale A: then the other factor on scale B will be adjacent to the product on scale A.

EXERCISE 1. Multiply 3 by 7, and 9 by 9.
Answers: 21 and 81.
2. Multiply 2·5 by 5·4, and 1·25 by 9·6.
Answers: 13·5 and 12.
3. What is the product of 3·85 by 7·62, and of 6·542 by 3·669? (See Note). Answers: 29·3 and 24.

PROBLEM II.

To divide one Number by another by Gunter's Scale.

RULE. Extend the compasses on the scale from 1 to the divisor. Keep the distance unaltered in the compasses, and it will extend backward from the dividend to the quotient.

EXAMPLE 1. What is the seventh part of 63?

Operation. Place the left foot of the compasses on 1 and the right on 7. Remove the compasses without altering the distance, and place the right foot on 63. The left will reach 9, which is the Answer.

COR. The same operation is performed by the *Sliding Rule*, thus:—Place the divisor on scale B adjacent to the dividend on scale A. The quotient will be on scale A adjacent to 1 on scale B.

Exercise 1. Divide 84 by 7, and 7·02 by 3·9.
Answers: 12 and 1·8.
2. Divide 7·9794 by 2·34, and 66·36 by 7·9.
Answers: 3·41, and 8·4.

Problem III.

To find the Fourth Term of a Proportion by Gunter's Scale.

Rule. Extend the compasses on the scale from the first term to the second. The same stretch will reach from the third to the fourth.

Or:—Extend the compasses from the first term to the third. That extent will reach from the second to the fourth.

Example 1. What number bears the same proportion to 5, that 12 bears to four?

Operation. As 4 : 12 :: 5 : 15.

Place the left foot on 4 and the right on 12. Keeping the same distance in the compasses, place the left on 5: the right will reach 15.

Or:—Place the left foot on 4 and the right on 5. That extent will reach from 12 to 15.

Example 2. Find the fourth term of the proportion, 21 : 27 :: 14 : ——.

Operation. As 21 : 27 :: 14 : 18. Ans. 18.

Cor. The same operation is performed *by the Sliding Rule*, thus:—

Place the first term on B adjacent to the second on A: then the third on B will be adjacent to the fourth on A. Or,

Place the first term on B adjacent to the third on A: then the second on B will be adjacent to the fourth on A.

Exercises. Find the fourth term of each of the following proportions :—

1. As 7 : 28 :: 5 : 20.
2. As 52 : 39 :: 64 : 48.
3. As 2·6 : 7·8 :: 3·2 : 9·6.

CHAPTER V.

DEFINITIONS AND EXPLANATIONS OF PLANE TRIGONOMETRY.

1. *Trigonometry* signifies literally *the Measurement of Triangles.*

2. Some triangles are delineated on a plane, others on a spherical surface; and, accordingly as Trigonometry relates to the former or to the latter class of triangles, it is distinguished by the names of *Plane* and *Spherical* Trigonometry.

3. *Plane Trigonometry* is not *exactly* what might be inferred from the etymology of its name. Properly explained, it is that department of Mathematics which treats of the practical computation of the sides and angles of plane triangles from each other. It does not include the computation of the areas of triangles, which belongs to Mensuration of Surfaces; and a few problems relating to the sides and altitudes of triangles, without reference to the angles, may be assigned either to Trigonometry or to Mensuration of Lines, more properly perhaps to the latter; while many theorems of similar reference are invariably assigned to Geometry.

4. The circumference* of every circle is supposed to consist of 360 equal parts, called *Degrees;* a degree, to consist of 60 *Minutes;* a minute of 60 *Seconds;* a second of 60 *Thirds,* &c.

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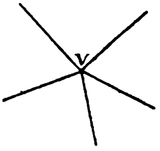
* With the common geometrical terms the scholar ought to be acquainted before commencing this volume.

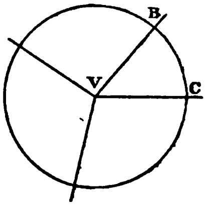
5. If any number of straight lines meet in a common point (V) they form a number of angles occupying all the angular space around that point. The whole of that angular space is supposed likewise to consist of 360 equal parts, called also Degrees, which are divided into Minutes, Seconds, &c., like the degrees of a circle.



Degrees are marked by a small circle over them; minutes by one accent; seconds, by two accents, &c.: thus $6^\circ\ 4'\ 14''\ 37'''$ means 6 degrees, 4 minutes, 14 seconds, 37 thirds.*

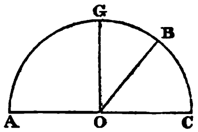
6. An arc or angle is thus measured by the number of degrees, minutes, &c., which it contains; and, in Trigonometry, when we speak of an angle, we commonly mean the number of degrees, &c., in the angle.

Corollary. If any number of lines meet in a point V, and a circle be described about that point as a centre, with any radius, then any angle, as BVC, is the same part of all the angles at V, that the arc BC is of the whole circumference: that is, each is the same part of 360°; or the angle BVC contains the same number of degrees as the arc BC. Hence, in the same circle, or in equal circles, angles at the centre are said to be measured by the arcs on which they stand.



7. The *Complement* of an arc or of an angle is what it wants of 90°. The *Supplement* is what it wants of 180°.

Thus the arc BG is the complement, and BA the supplement, of the arc BC; and the angle BOG is the complement, and BOA the supplement, of the angle BOC.



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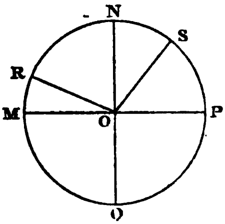
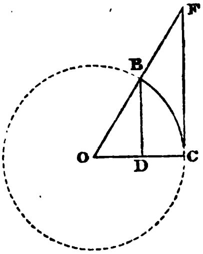
* Minutes are frequently divided decimally, and sometimes the degrees themselves are so divided, thus dispensing, in the former case, with the use of seconds, thirds, &c., and, in the latter case, with that of minutes also.



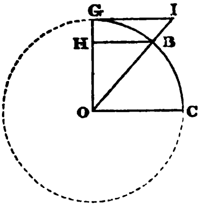
Exercise. In the following diagram let the scholar point out what is the complement of the arc SP, and what is its supplement. The same of the arc NS: then of the arcs MR and RN. He may then go over the same process with the angles at the centre.

8. Let BC be an arc of a circle and O its centre. Let OB, OC, be radii drawn to its extremities: from B and C let BD and CF be drawn perpendicular to OC and both terminated by the lines* of the two radii. Then BD is called the *Sine* of the arc BC; DC is its *Versed Sine;* CF, its *Tangent;* and OF its *Secant.*



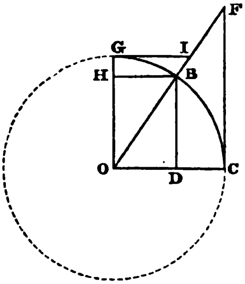
Exercise. Let the scholar draw a diagram like the first on this page, and exhibit, upon it, the sine, the versed-sine, the tangent, and the secant, first of the arc SP, then, successively, of the arcs NS, RN, and MR.

9. Returning to the arc BC, let BG be its complement. Then BH, the sine of the complement, is called the *Cosine* of the arc BC: GH, the versed-sine of the complement, is called the *Co-versed-sine* of BC: and, in like manner, GI is called the *Cotangent,* and OI the *Cosecant* of BC.



Exercise. In the annexed diagram let the scholar point out the sine, versed-sine, tangent, and secant of the arc BC; then the sine, &c., of the arc BG; then the cosine, co-versed-sine, cotangent, and cosecant, of the arc BC; then the cosine, &c., of the arc BG.

10. Since OD is = BH, OD also is called the cosine of the arc BC.



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* By the expression *lines of the radii*, is meant *the radii produced as far as may be necessary either way.*

11. The sine, tangent, &c., of the arc BC, are often called the sine, tangent, &c., of the angle BOC, to the radius OC.

12. Sometimes also they are called the sine, tangent, &c., of the number of degrees contained in the arc BC or in the angle BOC, the radius being at the same time named or understood.

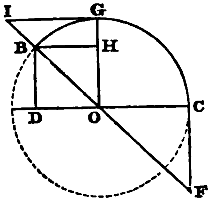
Thus, if the arc BC, or the angle BOC, is of 50°, and the radius OC is 10 of any equal parts, then BD is called the sine of 50°, CF the tangent of 50°, &c., to the radius 10.

. Exercise 1. If the radius is 10, what is the sine ot 90°; what is the chord of 60°; what, the sine of 30°; what, the cosine of 60°; and what, the versed-sine of 60°?

2. If the radius is 1, what is the tangent of 45°; what, the cotangent of 45°; what, the secant of 60°; what, the cosine of 0°; and what, the tangent of 90°?

3. The radius being 1, compute the sine and tangent of 60°, and the secant of 45°.*

13. When the arc BC is greater than a quadrant, the sine BD, the versed-sine DC, the tangent CF, the secant OF, the cosine BH or OD, the co-versed-sine HG, the cotangent GI, and the cosecant OI, occupy the places shown in the annexed diagram.



Cor. The sine of an arc is equal to the sine of its supplement.

Exercise. What is the sine of 30°; what, of 150°; what, of 60°; and what, of 120°; the radius being 100?

14. For use in calculation, the sines, tangents, secants, and versed-sines, of arcs, for every minute in the quadrant, to the radius 1, have been computed and arranged in tables; and, on the discovery of logarithmic arithmetic, the logarithms of the sines, tangents, secants, and versed-

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* The scholar is supposed to be acquainted with the first principles of Practical Geometry and of Mensuration, and nothing more is required for the above Exercises. The answers, when involving decimal fractions, may be carried out as far as the teacher pleases.

sines, for every minute of the quadrant, to the radius 10,000;000,000, were arranged in similar tables. The latter are called *Logarithmic Sines, Tangents*, &c.: they may be found in Table III of this volume. In contradistinction to these, the former are called *Natural Sines, Tangents*, &c.; but such is the facility which logarithms afford, that the natural sines, &c., have now been almost superseded by the logarithmic, and are seldom used.

15. The words *sine, tangent, secant, versed-sine, cosine, cotangent, cosecant*, and *co-versed-sine*, are often abbreviated thus: *sin, tan, sec, versin, cos, cot, cosec*, and *coversin.*

CHAPTER VI.

ON THE USE OF THE TABLES OF SINES, TANGENTS, AND SECANTS.*

PROBLEM I.

To find, from Table III, the Logarithmic Sine, Tangent, Secant, Cosine, Cotangent, or Cosecant of an acute Angle † *of any given number of Degrees and Minutes.*

RULE. *If the given number of degrees is less than* 45, look for the degrees in the top line of the table, turning over the leaves till the proper page is found. In that page, look in the second line for the name of the column wanted; and on the left margin of the page for the given number of minutes.

But, *if the given number of degrees is not less than* 45, look for the degrees in the bottom line of the page; for the name of the column, in the second line from the bottom; and for the minutes, on the right margin.

In either case, having found the minutes, then, in the same line, in its proper column, will be found the logarithm wanted.

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* These, of course, also include cosines, cotangents, and cosecants, which are merely sines, tangents, and secants of the complemental arcs or angles.

† Or of an arc less than a quadrant. But it is properly angles and not arcs on which Trigonometry is employed. Some given radius is understood, which is, of course, the radius of the table used.

Exercise 1. What is the logarithmic sine of 9° 10′ ?
Ans. 9·202234.

2. What is the log. tangent of 37° 26′ ?
Ans. 9·883934.

3. Find the log. secant of 64° 19′. Ans. 10·363114.

4. Find the log. cosine of 88° 44′. Ans. 8·344504.

Problem II.

To find, from Table III, the Logarithmic Sine, Tangent, or Secant of an Acute Angle of any given number of Degrees, Minutes, and Parts of a Minute.

Rule. Neglect the parts of a minute; but, if they amount to more than half a minute, increase the number of minutes by a unit. Then proceed as in the last problem.

Example 1. What is the logarithmic sine of 22° 32′ 48″ ?

Log sine of 22° 33′…………9·583754, Ans.

Example 2. What is the log tangent of 28° 47⅔′ ?

Log tan 28° 48′…………9·740169, Ans.

Example 3. Find the log. secant of 56° 22·38′.

Log sec 56° 22′…………10·256587, Ans.

Cor. *The logarithmic cosine, cotangent, and cosecant* may be found in the same manner as the sine, tangent, and secant.

Example 4. Find the logarithmic cosine of 26° 13·55′.

Log cos 26° 14′…………9·952793, Ans.

Exercise. Find the logarithmic tangent of 33° 17′ 57″; the log. sine of 68° 15⅜′; and the log. secant of 49° 28·84′.
Answers: 9·817484; 9·967927; and 10·187308.

PROBLEM V.

To find the Logarithmic Sine, Tangent, Secant, Cosine, Cotangent or Cosecant of a given Obtuse Angle.

RULE. Find that of its supplement.

EXAMPLE. What is the logarithmic sine of 129° 37′?

$$\begin{array}{l} 180^\circ\ 0' \\ 129^\circ\ 37' \\ \hline \end{array}$$

Log sine of 50° 23′............9·886676, Ans.

EXERCISE. What are the logarithmic sines of 170° 50′, and of 91° 16′? Answers: 9·202234, and 9·999894.

PROBLEM VII.

To find, from Table III, the Number of Degrees and Minutes, corresponding to any given Logarithmic Sine, Tangent, or Secant.

RULE. Look for the given sine, tangent, or secant, in its proper column; and, having found it exactly (or, if not exactly, the nearest to it whether larger or smaller), the minutes will be found on the same line on either the right or the left margin, and the degrees at the top or bottom of the page.

That is, if the name, "sine," "tangent," or "secant," is found at the head of the column, then the degrees are to be taken from the top of the page, and the minutes from the left margin: but, if the name is found at the foot of the column, then the degrees are to be taken from the bottom of the page, and the minutes from the right margin.

COR. The degrees and minutes corresponding to a given logarithmic *Cosine*, *Cotangent*, or *Cosecant*, are found in the same manner.

NOTE. It will depend upon the nature of the question whether the angle required is that found from the table or its supplement, except in the case of the versed-sine.

Exercise 1. What is the number of degrees and minutes corresponding to each of the logarithmic sines, 9·731009; 9·871073; 9·556978; and 9·943939?

Answers: 32° 34′; 48°; 21° 8′; and 61° 31′.

2. What are the degrees and minutes answering to the log. tangent 10·047850; to the log. tangent 9·787000; and to the log. secant 10·043700?

Answers: 48° 9′; 31° 29′; and 25° 16′.

3. What are the degrees and minutes answering to the log. cosine 9·862386; log. cotangent 9·787954; and log. cosecant 10·307796?

Answers: 43° 15′; 58° 28′; and 29° 29′.

CHAPTER VII.

OF RIGHT-ANGLED TRIANGLES.

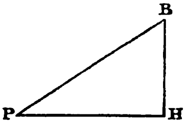
GENERAL NOTE.

Throughout this chapter, b represents the base; p, the perpendicular; h, the hypothenuse; and B, P, and H, the angles respectively opposite the sides b, p, and h.

Problem I.

In a right-angled Triangle, having given one of the two acute Angles, to find the other.

Rule. Subtract the given angle from 90°. (See Def. 6 in Ch. v.)



Formulæ. $\left\{\begin{array}{l} \angle P = 90° - \angle B. \\ \angle B = 90° - \angle P. \end{array}\right.$

Exercise 1. One of the acute angles of a right-angled triangle is 37½°: the other is required. Ans. 52½°.

2. What is the acute angle at the vertex when that at the base is 79° 48′? Ans. 10° 12′.

3. What is the acute angle at the base when that at the vertex is 54° 28·385′? Ans. 35° 31·615′.

4. The acute angle at the base being 11° 54′ 8″ 38‴, what is the angle opposite the base?

Ans. 78° 5′ 51″ 22‴.

PROBLEM II.

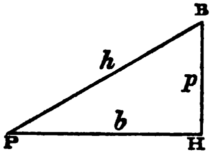
Having given the Hypothenuse and the Angles of a right-angled Triangle, to find the Legs.

RULE. As the radius* is to the sine of one of the acute angles, so is the hypothenuse to the leg opposite the said angle.

FORMULÆ.†

$\text{Radius} : \sin \text{P} :: h : p$

$\text{Radius} : \sin \text{B} :: h : b.$



NOTE 1. Instead of the sine of the angle opposite the required side, we may use the cosine of the acute angle adjacent to it, when the latter angle, and not the former, is given: it will save the trouble of subtracting. But it will be perceived that, in the table, the two are identical, the subtraction being performed, when there are no seconds, by merely looking from the left margin to the right for the minutes, and from the top of the page to the bottom for the degrees.

NOTE 2. It will be useful to the scholar, throughout all the exercises in Trigonometry, to draw the diagrams, marking the given sides and angles with their numerical values. It will also furnish useful exercises of Practical Geometry to draw such diagrams correctly, from a scale, and to ascertain from the same scale the required parts. The geometrical process should be a proof of the arithmetical. The diagram to the next example will illustrate this; but in future it will be left to the student to draw and mark the figure for himself.

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* By this is meant the radius used in the tables employed. If the table of natural sines is used, then the radius is 1; if the logarithmic table, the radius is 10,000,000,000, and its logarithm is 10.

† One formula is sufficient, since either leg may be made the base; but the double mode of expression will be clearer to beginners.

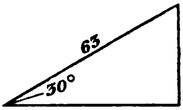
Example 1. The hypothenuse of a right-angled triangle is 63, and one of the acute angles 30°. What is the side opposite that angle?

By Chap. II, Problem XII,

As radius...............10·000000

Is to sin 30° .... 9·698970
So is 63 .... 1·799341

To ——— .... 1·498311.



Opposite this logarithm, in the column of sines in Table II, we find 31·5 as before.

Example 2. The acute angle at the base is 18° 9′ 42″, and the hypothenuse is 0·69848. The base is required by logarithms.

As radius...............10·000000

Is to cos 18° 10′ ....9·977794 (See Note 1, page 37).
So is 6·985..............0·844166 (See Note 3, page 16).

To 6·637 – .....0·821960

Ans. 0·6637 –.

Exercise 1. The hypothenuse of a right-angled triangle is 10·47, and the acute angle at the base is 58° 20′. Compute the length of the perpendicular. Ans. 8·91 +.

2. The base of a right-angled triangle is required, the hypothenuse measuring 89 yds. 2 ft., and the acute angle at the vertex 35°. Ans. 51 yds. 1 ft. +.

3. The acute angle at the base is 67¼°, and the hypothenuse 4·913: what is the base? Ans. 1·900.

4. The diagonal of a rectangular field is 344 links, making an angle of 22° 14′ with the longer side: what are the sides? Ans. 130·2 – and 318·4 +.

5. The hypothenuse is 0·396,* and one of the acute angles 15° 28′: what is the opposite side?

Ans. 0·1056 +.

6. The hypothenuse is 98·324,† and one of the acute angles is 48° 15·8′: ‡ calculate the opposite side.

Ans. 73·4 –.

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* See Chapter II, Problem XII, Note 3.
† See Chapter II, Problem VI.
‡ See Chapter VI, Problem II.

7. The hypothenuse is 0·48285,* and one of the acute angles 39° 42′ 15″: † what are the legs?

Answers: 0·308 +, and 0·371 +.

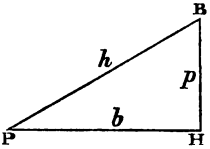
PROBLEM III.

In a right-angled Triangle, having given the Angles and one Leg, to find the Hypothenuse.

RULE. As radius is to the secant of the acute angle adjacent to the given side, so is the given side to the hypothenuse.

FORMULÆ.

Radius : sec P :: b : h.
Radius : sec B :: p : h.



NOTE. Instead of the secant of either of the acute angles, we may, if we please, use the cosecant of the other acute angle. (See Note 1 to Problem II.)

EXAMPLE. The base of a right-angled triangle is 29·42, and the acute angle at the vertex is 58° 30′: what is the hypothenuse?

$$\angle P = \text{compl. of } \angle B = 31° 30'.$$

As radius......................10·000000

Is to sec (P = 31° 30′).......10·069234
So is 29·42..................... 1·468643

To $h = 34{\cdot}50 +$.............. 1·537877.

EXERCISE 1. The base of a right-angled triangle is 31·5, and the adjacent acute angle is 60°: the hypothenuse is required. Ans. 63.

2. What is the hypothenuse when the perpendicular is 1900, and the vertical angle 67° 15′? Ans. 4913 +.

3. The base is 272·8, and the acute angle at the vertex 60°: what is the hypothenuse? Ans. 315, very nearly.

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* See Chapter II, Problem VI.

† See Chapter VI, Problem II.

4. The perpendicular is 78·324, and the acute angle at the base 28° 49′ 45″: what is the hypothenuse? Ans. 162·4 +.

5. The base is 0·076, and the vertical angle 14° 37·12′: what is the hypothenuse? Ans. 0·301 +.

PROBLEM IV.

In a right-angled Triangle, having given the Angles and one Leg, to find the other Leg.

RULE. As the radius is to the tangent of the acute angle adjacent to the given leg, so is the given leg to the other leg.

FORMULÆ. $\left\{\begin{array}{l} \text{Radius} : \tan P :: b : p. \\ \text{Radius} : \tan B :: p : b. \end{array}\right.$

(See the diagram on the following page.)

The second formula is the same as the first, applied to the other angle.

NOTE. Instead of the tangent of the acute angle adjacent to the given leg, we may take the cotangent of the angle opposite that leg.

EXAMPLE. The base of a right-angled triangle is 31·5, and the acute angle at the base 60°: the perpendicular is required.

As radius.............. 10·000000

Is to tan 60°........... 10·238561
So is 31·5.............. 1·498311

To Ans. 54·56 −...... 1·736872.

EXERCISE 1. The base of a right-angled triangle is 69·4; and the acute angle at the base, 45°: what is the perpendicular? Ans. 69·4.

2. The perpendicular is 4,984, and the vertical angle is 33° 17′: what is the base? Ans. 3272 −.

3. The acute angles of a right-angled triangle are 35 and 55 degrees. The leg opposite the latter is 9 feet 5 inches: the other leg is required. Ans. 6 ft. 7 in.

4. Base, 94·738; adjacent angle, 48° 23′ 10″: find the perpendicular. Ans. 106·6+.

5. Perpendicular, 0·106654; vertical angle, 41° 36·83′: find the base. Ans. ·0947+.

PROBLEM V.

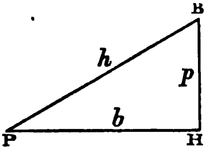
In a right-angled Triangle, having given the Hypothenuse and one Leg, to find the Angles.

RULE 1. As the hypothenuse is to the given leg, so is the radius to the sine of the angle opposite the given leg.

FORMULÆ.

$h : p :: \text{radius} : \sin P.$

$h : b :: \text{radius} : \sin B.$



RULE 2. The one leg is to the hypothenuse, as the radius to the secant of the angle adjacent to the said leg.

FORMULÆ. $\begin{cases} b : h :: \text{radius} : \sec P. \\ p : h :: \text{radius} : \sec B. \end{cases}$

NOTE. Having found the one angle, the other is found by Problem I. Or, if we choose, we get the other at once, by taking the fourth term of the proportion as a cosine or a cosecant instead of a sine or a secant.

EXAMPLE. The hypothenuse is 63, and the perpendicular 31·5: the angles are required.

By Rule 1.

As 63 .... 1·799341

Is to 31·5 .... 1·498311
So is radius........10·000000

To sin 30° .... 9·698970.

Ans. ∠ P = 30°. ∴ ∠ B = 60°.

By Rule 2.
As 31·5 .... 1·498311
Is to 63 .... 1·799341
So is radius........10·000000
To sec 60°.........10·301030.

Ans. $\angle B = 60^\circ$ $\quad \therefore \angle P = 30^\circ$.

Exercise 1. The hypothenuse is 10·47, and the perpendicular 8·91: find the acute angle at the base.
Ans. 58° 19′+.

2. What is the angle at the vertex, the hypothenuse being 89 yds. 2 ft., and the base 51 yds. 1 ft.?
Ans. 34° 55′+.

3. The base is 1900, and the hypothenuse 4913: what is the acute angle at the base? Ans. 67° 15′−.

4. The hypothenuse being 5·235, and the perpendicular 4·455, the vertical angle is required. Ans. 31° 41′−.

5. Hypothenuse, ·034; base, ·028: the angles are wanted. Answers: 34° 34′−, and 55° 26′+.

6. Hypothenuse, 16,386; perpendicular, 9,569: find the angles. Answers: 54° 17′−, and 35° 43+.

Problem VI.

In a right-angled Triangle, having given the Legs, to find the acute Angles.

Rule. As either leg is to the other leg, so is radius to the tangent of the angle opposite the latter.

Formulæ.
$$\begin{cases} b : p :: \text{radius} : \tan P. \\ p : b :: \text{radius} : \tan B. \end{cases}$$

(*See the diagram on the preceding page.*)

Example. The base is 3, and the perpendicular 4: what are the angles?

We may either say,

As 3 : 4 :: radius : tan P, or
As 4 : 3 :: radius : tan B.

We take the former.

As 3 .... 0·477121

Is to 4 .... 0·602060
So is radius..............10·000000

To tan 53° 8′............10·124939.

Ans. $\angle P = 53^\circ\, 8'$ $\therefore$ $\angle B = 36^\circ\, 52'$.

Exercise 1. The two legs are 31·50 and 54·56: the angles are required. Ans. 30° and 60°, nearly.

2. The base is 6·94, and the perpendicular the same: what are the angles? Ans. Each 45°.

3. The perpendicular is 4,984, and the base, 3,272: what is the vertical angle? Ans. 33° 17′ +.

4. The perpendicular is 9 ft. 5 in., and the base 6 ft. 7 in.: what is the acute angle at the base. Ans. 55°, nearly.

5. Given, the legs ·07859 and ·06990: required the angles. Ans. 41° 39′ + and 48° 21′ −.

6. The legs are 20,671 and 9,473·8: the angles are wanted. Ans. 24° 37′ + and 65° 23′ −.

GENERAL NOTE.

If it be asked how the preceding rules are to be *kept in memory* without reference to the book, observe the following direction :—

Suppose a circle to be described with a given side (whether leg or hypothenuse) as radius. Then observe what the other given or required side becomes with reference to the circle just described,—that is, whether the sine, tangent, or secant; and write that on the last-mentioned side as its name. The name of that side will thus be the sine, tangent, or secant, of the opposite angle.

This being done, if one of the three sides is required, say :—As the *radius* of the table, to the *name* written on the other side, so is the side used as radius, to the other side.

But, if an angle is required, say :—As the *side* used as a radius, is to the other given *side*, so is the radius of the table to the name written on the other side.

This mode of proceeding is what is usually called "*Making a given side Radius.*"

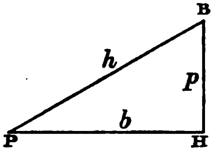
Problem VII.

In a right-angled Triangle, having given the Hypothenuse and one Leg, to find the other Leg.

Rule 1. Subtract the square of the given leg from the square of the hypothenuse, and extract the square root of the remainder.

FORMULÆ.

$$b=\sqrt{(h^2-p^2)}.$$
$$p=\sqrt{(h^2-b^2)}.$$



For Examples of the application of this rule and of Rule I of the next problem, as well as for numerous Exercises in both, see Part II, Ch. III, Pr. I. The scholar may also, if he choose, work the following exercises by this rule.

Rule 2. Multiply the sum of the given leg and hypothenuse by their difference, and extract the square root of the product.

Formulæ. $$\left\{\begin{array}{l} b=\sqrt{\{(h+p)\times(h-p)\}}.\\ p=\sqrt{\{(h+b)\times(h-b)\}}.\end{array}\right.$$

Example 1. The hypothenuse is 654 and the perpendicular 583 : what is the base ?

654
583
—— Logarithms.
Sum = 1237...............3·092370
Difference = 71...............1·851258

2)4·943628

Answer, 296·4 – ....2·471814.

Rule 3. Find the acute angles by Problem V, and then the required leg by Problem II.

Note 1. It is not necessary to find the angles themselves, but merely their sines (natural or logarithmic): for, having obtained the sine of the one acute angle, and found the nearest to it on the table, we find the sine of the other acute angle (when great accuracy is not required) by merely looking along the line from the right hand column of sines to the left, or from the left to the right.

Note 2. After finding the angles, we may, if we choose, use Problem IV, instead of Problem II, for finding the required leg.

Example 2. Find the answer to Example I by Rule III.

As 654………	2·815578	As radius…….	10·000000
Is to 583…….	2·765669	Is to sin ∠B…	9·656302
So is radius,…	10·000000	So is 654…….	2·815578
To sin ∠P…..	9·950091	To $b = 296{\cdot}4 +$	2·471880

Ans. 296·4 +.

Exercises in Rules 2 and 3. 1. The hypothenuse is 57·97, and the perpendicular 51·15: compute the base. Ans. 27·3 –.

2. The hypothenuse is 617·2, and the base 493·8: compute the perpendicular. Ans. 370 +.

3. The hypothenuse is 2·4286, and the one leg 2·1429: what is the other leg? Ans. 1·143.

4. The hypothenuse being 0·126731, and the one leg ·027819, what is the other leg? Ans. 0·124 –.

Problem VIII.

In a right-angled Triangle, having given the two Legs, to find the Hypothenuse.

Rule 1. Add together the squares of the two legs, and extract the square root of the sum.

Formula. $h = \sqrt{(b^2 + p^2)}$.

Rule 2. Find either acute angle by Problem VI, and then the hypothenuse by Problem III.

Note. See Note 1 to the last Problem.

Example. The base is 0·24915, and the perpendicular 0·59796 : what is the hypothenuse?

As 2·491...... 0·396374	As radius.......10·000000
To 5·980.......0·776701	To sec P........10·415032
So is radius...10·000000	So is 2·491..... 0·396374
To tan P......10·380327.	To 6·477 + ... 0·811406.

Ans. 0·648 – .

Exercises in Rule 2. 1. The base is 27·28, and the perpendicular 51·15 : determine the hypothenuse. Ans. 58, very nearly.

2. The two legs are 493·76 and 370·32 : the hypothenuse is required. Ans. 617 + .

3. The legs are 1·1429 and 2·1429 : find the hypothenuse. Ans. 2·429.

4. What is the hypothenuse if the legs are 0·12364 and ·027819? Ans. 0·127 – .

Promiscuous Exercises in Right-angled Triangles.

1. The base is 6 ; and the acute angle at the base, $67\frac{3}{4}$° : what is the hypothenuse, and the perpendicular? Ans. 15·85 –, and 14·67 – .

2. The hypothenuse is 3,965 ; and the base, 3,172 : what is the perpendicular? Ans. 2,379.

3. The hypothenuse is 100, and one of the acute angles four times the other : what are the sides? Ans. 30·90 +, and 95·11 – .

4. The one leg of a right-angled triangle is four times the length of the other : what are the acute angles? Ans. 75° 58′ –, and 14° 2′ + .

5. Each of the sides of an isosceles triangle is four times its base : what are its angles? Ans. 14° 22′ –, 82° 49′ +, and 82° 49′ + .

6. The diameter of a circle is 1,000 : what is the chord of an arc of 30°? Ans. 258·8 + .

7. What is the altitude of an equilateral triangle whose side is ·08? Ans. ·06928 + .

8. The chord of an arc is 0·895; and the chord of its supplement 2·148: what is the diameter of the circle? Ans. 2·327.

9. The sine of an arc is 88, and the cosine 89: how many degrees are in the arc, and what is its radius? Ans. 44° 41′, and radius 125 + .

10. The hypothenuse is 64·779, and the perpendicular 59·796: what is the base? Ans. 24·9 + .

CHAPTER VIII.

OF OBLIQUE-ANGLED TRIANGLES.

GENERAL NOTE.

Throughout this chapter, *b* represents the base of the triangle; and *a* and *c*, the other two sides; while A, B, and C are the three angles respectively opposite the sides *a*, *b*, and *c*, as marked in the following diagram.

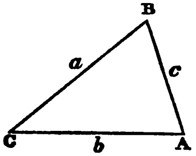
Problem I.

In any Triangle, having given two of the Angles, to find the third Angle.

Rule. Subtract the sum of the two given angles from 180 degrees.

FORMULÆ.

$$A = 180° - (B + C).$$
$$B = 180° - (A + C).$$
$$C = 180° - (A + B).$$



Exercise 1. Two angles of a triangle are respectively of 48½ and 92¼ degrees: of how many degrees is the third angle? Ans. 39¼.

2. The angle A is 38° 24′ ; the angle B, 56° 31′ : what is the angle C ? Ans. 85° 5′.

3. The angle A is 44° 5′ 49″ ; C is 62° 15′ 28″ : what is B ? Ans. 73° 38′ 43″.

4. What is the vertical angle of an isosceles triangle, each of the angles at the base being of 43° 47·85′ ? Ans. 92° 24·3′.

5. The angle at the vertex of an isosceles triangle is 56° 24½′ : what is one of the angles at the base ? Ans. 61° 47¾′.

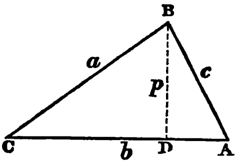
PROBLEM II.

In any Triangle, having given the Angles and one Side, to find either of the other Sides.

Rule. As the sine of the angle opposite the given side to the sine of the angle opposite the required side, so is the given side to the required side.

FORMULA.

Sin A : sin C :: a : c.



a being the given side, and A its opposite angle ; c, the required side, and C its opposite angle.

Note. The greater side is always opposite the greater angle, and the less side opposite the less angle. It must also be kept in mind that the sine of an angle of more than 90° is the same as the sine of its supplement.

Example. Given :—two angles of a triangle 36° 45′ and 79° 34′ ; and the side opposite the former 4834. Required :—the side opposite the latter.

As sine 36° 45′..............9·776937

To sine 79° 34′..............9·992759
So is 4834.....................3·684307

To Ans. 7946 – ....3·900129.
280

Exercise 1. Two angles of a triangle are 58° 12′, and 64° 33′; the side opposite the former is 385: what is the side opposite the latter? Ans. 409 +.

2. The three angles of a triangle are 38° 24′, 46° 31′, and 95° 5′; the greatest side is 7·832: what is the least side? (See the Note). Ans. 4·884 +.

3. In the triangle described in the last exercise, what is the third side? Ans. 5·705 +.

4. Two of the angles of a triangular field being 63° 55′ and 48° 27′, and the side opposite the latter 5¾ chains: what are the other two sides?

Ans. 6·901 −, and 7·105 + chains.

5. Each of the equal angles at the base of an isosceles triangle is 61° 47′, and the base 39 ft. 5 in.: what are the sides? Ans. Each 41 ft. 8 in. +.

6. Two of the angles of a triangle are 35° 12′, and 116° 33′; the side opposite the former is 0·5876: what are the other two sides?

Ans. 0·9119 −, and 0·4825 −.

7. One side of a triangle measures 784·36 feet, and the opposite angle 64° 31·25′: what is the side opposite a second angle of 91° 4·75′? Ans. 869 − feet.

PROBLEM III.

In any Triangle, having given two Sides and an Angle opposite one of them, to find the other Angles.

Rule. As the side opposite the given angle is to the other given side, so is the sine of the given angle to the sine of the angle opposite the latter side.

This being found, the third angle is obtained by Problem I.

Formula. $a : c :: \sin A : \sin C$.

A, C, a, and c, being the same as in the last problem.

The rules for *both* problems are expressed by the following **Theorem**:—*The Sides of any Plane Triangle are to each other as the Sines of the opposite Angles.*

Note. Since, by the rule, we find the *sine* of the required angle, and not the angle itself, and since the sine of any angle and that of its supplement are the same, we always obtain two values for the angle found by the rule,—one less than 90°, the other greater: some directions are therefore necessary, to know which of the two values should be taken.

1. When the form of the triangle is previously known, we take either the smaller or the greater value, according as the angle is acute or obtuse.

2. When the given angle is obtuse, the angle found must be acute, since no triangle can have two obtuse angles.

3. When the given angle is acute, but opposite the greater of the two given sides, the angle found must be less than the given angle, and therefore acute also.

4. When the given angle is acute, and opposite the smaller of the two given sides, and when, at the same time, the form of the triangle is unknown, we may take either of the two values (or rather, we must take both), since either will suit the conditions of the question.

Example. Two sides of a triangle are 0·4834 and 0·7946, and the angle opposite the former is 36° 45′: what are the other angles?

As 4·834 (*a*)	0·684307
To 7·946 (*c*)	0·900149
So is sine 36° 45′ (A)	9·776937
To sine C	9·992779.

Looking for the last logarithm in the column of sines, in Table III, we find it opposite 79° 35′ −. But, since the sine of an angle is the same as that of its supplement, the angle C may be either 79° 35′ −, or 100° 25′ +. Then, by Problem I, we find the angle B = 63° 40′ +, or 42° 50′ −.

Exercise 1. Two sides of an acute-angled triangle are 409 and 385; the angle opposite the former is 64° 32′: what is the angle opposite the latter? Ans. 58° 12′ −.

2. In a triangle, obtuse-angled at the vertex, the one side is 4·861, and the base is 8·583; the angle opposite the given side is $23\frac{3}{4}$°: what is the vertical angle?

Ans. 134° 40′ +.

3. The three sides of a triangle are 4·884, 7·832, and 5·692; the greatest angle is 95° 15′: what are the other angles? Ans. 38° 23′ +, and 46° 22′ −.

4. Two sides of a triangle are 0·33 and 0·44; the angle opposite the latter is 78° 14′: what is the angle opposite the former? Ans. 47° 15′ −.

5. Two sides of a triangle being 6·9014 and 5·7498, and the angle opposite the latter 48° 27·2′: what are the other angles?

Ans. 63° 55′ +, or 116° 5′ −; and 67° 38′ −, or 15° 28′ ±.

PROBLEM IV.

In any Triangle, having given two Sides and an Angle opposite one of them, to find the third Side.

RULE. Find the angles by Problem III, and then the third side by Problem II.

NOTE. When the angles found have each a double value (as explained in the Note to the last problem), the third side also has a double value.

EXAMPLE. Two sides (*a* and *c*) of a triangle are 483·4 and 794·6; the angle A (opposite the side *a*) is 36¾°: what is the third side?

The angle B, found as in the last example, is 63° 40′ +, or 42° 50′ −. Then

As sine (36° 45′ = A).........9·776937......9·776937

To sin 63° 40′ or 42° 50′...9·952419, or 9·832425
So is (483·4 = *a*)............2·684307......2·684307

To Ans. 724, or 549 ....2·859789......2·739795.

EXERCISE 1. Two sides of an acute-angled triangle are 409 and 385; the angle opposite the former is 64° 32′: what is the third side? Ans. 381 +.

2. In a triangle, obtuse-angled at the vertex, the base is 8·583, and the one side 4·861; the angle opposite the said side is 23° 45′: what is the other side?

Ans. 4·44 −.

3. The greatest side of a triangle is 7·832, and the least side 4·884; the greatest angle is 95° 15′: compute the third side. Ans. 5·69 +.

4. Two sides of a triangle are 3 and 4; the angle opposite the latter is 78° 14′: what is the third side? Ans. 3·33 −.

5. Two sides of a triangle being ·069014 and ·057498, and the angle opposite the latter being 48° 27·2′, what is the remaining side? Ans. ·0711 −, or ·0205 −.

Problem V.

In any Triangle, having given two Sides and the included Angle, to find the other Angles.

Rule. Subtract the given angle from 180°: the remainder will be the sum of the two required angles. Then say,—

As the sum of the two given sides is to their difference, so is the tangent of half the sum of the two required angles, to the tangent of half their difference.

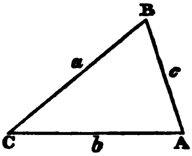
Having found the said half difference, add it to the half sum of the same two angles (previously found) for the greater angle, or subtract it from the half sum for the smaller angle.

FORMULÆ.

$$\angle A + \angle C = 180° - \angle B.$$

$$a + c : a - c :: \tan \tfrac{1}{2}(A + C) : \tan \tfrac{1}{2}(A - C).$$

$$A = \tfrac{1}{2}(A + C) + \tfrac{1}{2}(A - C).$$
$$C = \tfrac{1}{2}(A + C) - \tfrac{1}{2}(A - C).$$



a and c being the given sides, B the given angle, and A and C the required angles.

Example. Two sides of a triangle (a and c) are 794·6 and 724·4 respectively; and the angle (B) contained between them is 36° 46′: what are the other angles?

180° 0′	794·6 = a
36 46 = ∠B	724·4 = c
2)143 14 = Sum of A and C	1519·0 = Sum.
71 37 = Half sum.	70·2 = Diff.

As 1519........................ 3·181558

Is to 70.2 .... 1·846337
So is tan 71° 37′.............10·478427

To tan 7 55 .... 9·143206

∠A = 79 32 } Answer.
∠C = 63 42 }

Exercise 1. Two sides of a triangle are 409 and 381, and the angle included between them is 58° 12′: what are the other angles?
Answers: 64° 33′ −, and 57° 15′ +.

2. The vertical angle of a triangle is 23° 46′, and the two sides 9·72 and 8·88: what are the two angles at the base? Answers: 90° 14′ −, and 66° 0′ +.

3. One angle of a triangle is 97° 26′, and the two sides which contain it are 0·843 and 0·657: what are the angles opposite those sides?
Answers: 47° 30′ −, and 35° 4′ +.

4. Two sides of a triangle are 7,103·5 and 6,901·4; their included angle is 48° 27·2′: what are the other angles? Answers: 67° 36′ +, and 63° 56′ +.

PROBLEM VI.

In any Triangle, having given two Sides and their included Angle, to find the third Side.

Rule. Find the other angles by Problem v, and then the third side by Problem ii.

Note. After finding the two unknown angles, only one of these and one of the two given sides is employed to find the third side. Of the said two angles, the greater should always be taken in preference to the smaller, and consequently, also, the greater of the two given sides. These give the more accurate result. But if the said angles and sides are not very unequal, the smaller may be used for a second computation in proof of the first. The true answer will always be between the two results, but nearest to that obtained by using the greater side and angle.

Exercise 1. Two sides of a triangle are 8·18 and 7·62, and their included angle is 58° 12′: what is the third side? Ans. 7·70 –.

2. The vertical angle of a triangle is 47° 32′, and the two sides 57·3 and 48·4: what is the base? Ans. 43·37.

3. Two sides of a triangle being 0·12 and 0·05, and including an angle of 90 degrees: compute the third side by this rule. Ans. 0·13.

4. Two sides of a parallelogram are 39·7 and 48·3, and each of its acute angles is 56° 14′: what are the diagonals? Answers: 77·7 +, and 42·2 –.

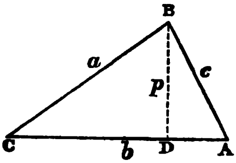
5. Find the base of a triangle which has its two sides 71·035 and 69·014, and the vertical angle 48° 27·2′. Ans. 57·5 –.

Problem VII.

In any Triangle, having given the three Sides, to find the Angles.

Rule. In an acute-angled triangle call any one of the three sides the base; but in an obtuse-angled triangle call the longest side the base. Draw a perpendicular, BD, from the vertex to the base, or suppose it drawn. Then say,—

As the base is to the sum of the two sides, so is the difference of the two sides, to the difference of the segments of the base.



Half the difference, thus found, added to half the base, will give the greater segment, CD; and half the difference, subtracted from half the base, will give the smaller segment, AD.

Having found these, the angles

C and A are found, by Problem v of the last Chapter; and then the angle B, by Problem I of this Chapter.

FORMULÆ.

$$b : a+c :: a-c : d.$$

$CD = \frac{1}{2}b + \frac{1}{2}d.$	$AD = \frac{1}{2}b - \frac{1}{2}d.$
$CD : a ::$ radius : sec C.	$AD : c ::$ radius : sec A.

d being the difference of the segments of the base.

Example. The three sides of a triangle are 28, 40, and 16: what are the angles?

By Common Arithmetic.

28	As 40 : 44 :: 12
16	— — 11
—	10 11 —
$44 = a+c$	10)132
—	——
$12 = a-c$	$13{\cdot}2 = d.$

By Logarithms.

As 40 ....1·602060	$20{\cdot}0 = \frac{1}{2}b.$
	$6{\cdot}6 = \frac{1}{2}d.$
Is to 44 ....1·643453	
So is 12 ....1·079181	$26{\cdot}6 = CD.$
To $d = 13{\cdot}2$............1·120574.	$13{\cdot}4 = AD.$

As CD = 26·6............... 1·424882

Is to $a = 28$.................. 1·447158
So is radius..................10·000000

To sec 18° 12′ − ....10·022276.

As AD = 13.4...... 1·127105	∠C = 18° 12′ −
	∠A = 33 7 +
Is to $c = 16$......... 1·204120	
So is radius.........10·000000	51 19
	180 0
To sec 33° 7′ + ...10·077015.	∠B = 128 41.

NOTE. The rule proceeds on the supposition that the two sides are unequal. If they are equal, the first part of the rule is evidently not required, since, in that case, the segments are each half the base.

EXERCISE 1. The three sides are 735, 595, and 350 feet: what are the three angles?

Ans. 28° 4′+, 98° 48′, and 53° 8′−.

2. What are the three angles, supposing the three sides respectively 0·532, 0·259, and 0·371 miles?

Ans. 39° 33′+, 26° 24′−, and 114° 3′−.

3. The base of a triangle measures 64 ft. 8½ in., and the two sides respectively 45 ft. 7½ in., and 36 ft. 3 in.: what are the angles?

Ans. 32° 57′+, 43° 12′+, and 103° 51′−.

CHAPTER IX.

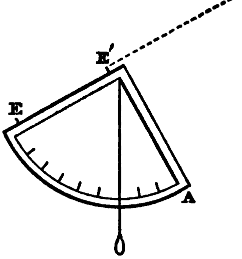
OF THE APPLICATIONS OF PLANE TRIGONOMETRY.

The principal applications of Trigonometry are to the measurement of *Heights and Distances* by means of angular observations, to *Navigation*, to *Astronomy*, and to *Mechanics and the other departments of Natural Philosophy*.

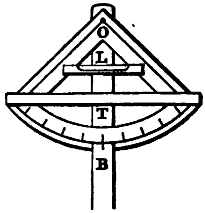
Trigonometry is much employed in *Land-surveying*, when conducted on an extensive scale, such as the National Survey of the British Isles, but the processes employed are the same as those for the determination of Heights and Distances. When a survey is spread over so large a portion of the Earth's surface that it cannot be regarded as a plane, the aid of Spherical Trigonometry then becomes essential. Questions relating to that class of operations do not belong to the subject of this volume.

Many works on Trigonometry describe various simple modes of ascertaining unknown heights and distances entirely independent of angular measurements, and include questions relating to these methods. Such questions belong more properly to *Mensuration of Lines*.

In the course of the following Exercises, allusion will frequently be made to instruments employed for the measurement of angles of observation,—that is, for determining the angular elevation or depression of any distant object above or below the horizon, or the angular distance of any two such objects from each other, the point of observation being taken as the centre or angular point. For the sake of perspicuity it is proper to explain here briefly the nature of each of these instruments and its appropriate uses.

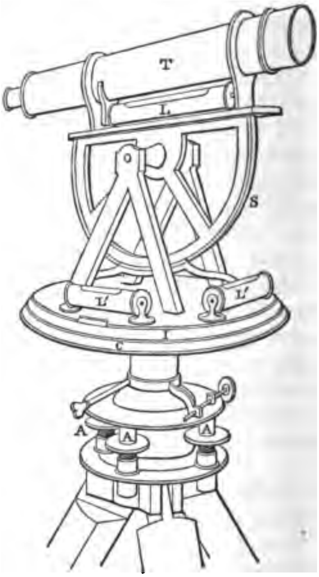


The first and simplest of these is the old-fashioned *Quadrant* represented in the annexed diagram. It is in the form of a sector of a circle, is commonly made of wood, and has its arc divided into ninety degrees commencing at the extremity A. A plumb line is suspended from the centre by a thread, and on the side opposite A are placed two sight vanes, E and E′: these are two pieces of brass with a small hole in each, or other mark, the said holes or other marks being in a line parallel to the adjacent radius. In taking angles of elevation we look from E to E′; but, in taking angles of depression, we look from E′ to E. The instrument is either held in the hand by the person taking the observation, while the degrees are read by an assistant, or is placed on a support, and, when properly directed (with the object in a line with the sights), is fixed in that position till the angle is read.

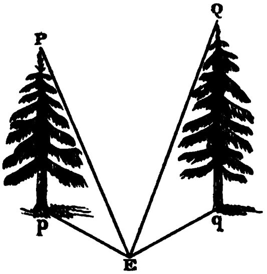


An improved form of the quadrant has a telescope, T, instead of the sight vanes, and a spirit level, L, to serve the same purpose as the plumb line, in a different manner. The degrees are numbered from B both ways, the instrument being limited to angles of not more than 45 degrees, whether of elevation or of depression. It is placed on a stand on which is a fixed index pointing out the number of degrees, while the quadrant turns on the fixed centre, O. The spirit level is attached either to the quadrant or to the fixed support. In the latter case it remains level during the operation: in the former it is level only when the index is at zero. In adjusting the instrument previously to the observation, the index is fixed at zero, the telescope is turned towards the object, and the stem is brought as nearly as possible to an upright position by means of the feet on which it stands; and, when that is done, a screw acting on the stem brings the spirit level to a horizontal position and completes the adjustment in the direction of the telescope, while that in the cross direction, being considered of comparatively little importance, is determined by the eye, and attained by the use of the feet of the instrument only. The exact point to which the telescope is directed is determined by two hairs or other fine lines, crossing the sight at right angles to each other, —one being horizontal, the other vertical.

The quadrant in either of the preceding forms is used only for vertical angles, that is, angles in a vertical plane. But even for that purpose it is an imperfect instrument, and is not employed in nice operations. In these it is superseded by the following:—



The *Theodolite* consists of a vertical semicircle, S, connected with a horizontal circle, C, both graduated at the circumference. The graduated horizontal circle remains fixed during an observation, while another concentric circular plate, I, turns upon it, having an index on its margin: but the vertical arc moves, while its index remains fixed. While the vertical semicircle and the horizontal index-circle turn freely, each on its own axis, the axes themselves are firmly connected with each other. Two spirit levels, L′, L′, placed on the circle, at right angles to each other, determine its accurate position in a horizontal plane, and, at the same time, the position of the semicircle in a vertical plane; the instrument being brought nearly into that position by shifting the feet on which it stands, and then adjusted exactly by the screws A, A. A telescope, T (having its sight crossed by two hairs, as before described), is fixed to the vertical semicircle parallel to its diameter, and, consequently, turning with it, so that the angular elevation or depression of the telescope is determined by the number of degrees passed over by the index at the circumference; the instrument, previously to any observation, being so adjusted (by means of a third level L), that, when the telescope is directed to an object in the horizon, the index of the vertical semicircle points to zero.* Again, when the telescope turns from one object to another, both in the horizon, it carries the index-circle round with it, the graduated circle showing the number of degrees of revolution. But when the telescope turns from one object to another, neither in the same vertical nor in the same horizontal plane with each other and with the place of observation, both the vertical semicircle and the horizontal index-circle turn on their axes; but, in that case, neither of them shows the angular distance between the two observed objects. The former merely shows the angular elevation or depression of each object above or below the horizon, while the latter shows the difference of their bearings or horizontal directions,—that is, the angular distance of the *seats* of the two objects on the horizontal plane of the point of observation. Thus, E being the point of observation, let p and q be two other points in the same horizontal plane with it, and let Pp and Qq be two perpendiculars to that plane. If the telescope at E turn from the object P to Q, the horizontal circle turns exactly the same number of degrees as if the telescope had been directed first to p and then to q; for the telescope would evidently pass from P to p, or from Q to q, without any movement of the horizontal circle. Therefore the horizontal circle measures the angle pEq, not the angle PEQ, although the telescope turns from P to Q. The latter angle, when



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* Or if not exactly, the error must be added or subtracted at each observation.

the theodolite is used, can only be obtained by calculation, but is, in reality, seldom required. This must be carefully borne in mind in computations of heights and distances.

The last instrument to be described is the *Quadrant* or *Sextant*, commonly called *Hadley's*, after its inventor.* The sextant and quadrant are, in reality, the same instrument, differing only in the extent of the graduated arc; the quadrant measuring angles of 90°, and the sextant those of 120°. When the whole circle is complete, the instrument becomes what is called the *Reflecting Circle;* being then made in the most perfect manner, and accompanied by several appendages, which render it altogether "a very refined and elegant instrument." The quadrant is never made with the same nicety of construction as the sextant, and the reflecting circle is used only when extreme accuracy is required: when, therefore, the instrument is alluded to, in the description and exercises which follow, I shall speak of it by the general name of sextant.

It is an instrument beautifully adapted, by means of a peculiar ingenious contrivance, to nautical use; and, since it is the only instrument of observation which can be used at sea, its value to the navigator is beyond estimation. Such is the instability of every vessel, from the restless motion of the water, as absolutely to prevent the use of any apparatus placed on a fixed support. Whatever mechanism is employed, must, it is obvious, be held in the hand. But to take the angular distance between any two objects, with an instrument so held, is evidently impossible, unless both objects can be viewed at the same instant, and, consequently, in the same direction. This is precisely what Hadley's contrivance effects. By reflecting mirrors it deflects one of the objects apparently from the direction in which it would otherwise be seen, and brings it into seeming close contact with the other object, so that both can be seen at the same moment; and we have only to keep the telescope of the sextant steadily upon the *one* object (which can be done, notwithstanding the motion), to see the other also; while the adjustment of the mirrors necessary to effect the apparent coincidence of the two objects, serves, at the same time, to show the angular distance of the objects from each other.

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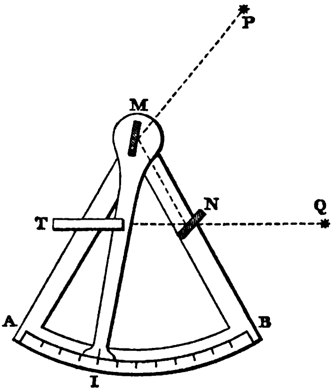
* Hadley first gave it to the world; but priority of invention is attributed to Sir Isaac Newton.

Thus, in the diagram, P and Q being the two objects, and M and N two small mirrors attached to the instrument, half of the glass forming the mirror N is left transparent, and the object Q is seen directly through the telescope T, and through the transparent half of the glass N, while the other object, P, is reflected first from the mirror M to the mirror N, in which it is seen, through the telescope, apparently close to Q, provided that the mirrors are suitably placed with regard to each other. The adjustment of the mirrors, with that view, is effected by turning the moveable radius MI, to which the mirror M is attached, until the apparent coincidence is effected. The degrees are then read on the arc AB, being pointed out by the index, I, on the moveable radius, MI.



The arc of the sextant is really of 60 degrees, but is divided into 120 equal parts, counted as degrees, and actually measuring degrees in the observed angles, since any movement of the index, I, deflects the object (P) from its apparent position, by twice as many degrees as those passed over by the index. In the same manner the arc of the quadrant is really of 45 degrees, but measures angles up to 90 degrees. The two instruments are not named analogically, the sextant taking its name from the actual arc of the instrument, the quadrant from the angle it is capable of observing.

While the theodolite is confined to vertical and horizontal angles, the sextant measures angles in any oblique plane. The latter instrument can be applied to the measurement of horizontal angles, as such, only when the objects observed are previously known to be both on the same level with the observer, as in the case of a survey of a harbour or of a sea shore, where signals can be placed at the same height, above the surface of the water, with the point of observation. For vertical angles, again, the sextant cannot be used without the aid either of the *sea-line* or of an instrument called an *artificial horizon.*

When the surface of the ocean is seen unbroken by land and bounded only by the sky, if the observer is raised but little above its surface, its outline nearly coincides with the horizon, and may, therefore, with a slight correction,* be taken as the horizon, the angular elevation of any object above the sea-line being taken, with that correction, as its elevation above the horizon. When the sea immediately below the object is bounded by land, its outline is still taken as the horizon, when its distance is known; but, in this case, a different correction† must be made for the depression of the observed outline, since it always appears lower than the sea-line would have appeared, had the prospect been unbounded.

When the sextant is used on land for taking angles of elevation, since the sea-line cannot then be used as a horizon, a small basin of mercury is taken in which the object is seen reflected. The angle between the object and its reflection is then observed, and half of that is taken as the angle of elevation, since the angle of reflection is always equal to the angle of incidence. To prevent agitation by the wind, the mercury is frequently covered by a roof of plate glass. The basin or box thus fitted up is called an *Artificial Horizon.* When mercury is not at hand, any other liquid presenting a reflecting surface may be used. It happens very conveniently that the artificial horizon is available precisely in those circumstances in which the sea-line cannot be employed, that is, in observations on land; and that, where the instability of the water renders the artificial horizon useless, it is peculiarly there that the outline of the sea comes in to supply its place.

In surveying, the sextant may be used when the actual distances only are required, without reference to heights or to horizontal dimensions. But if the heights are required, or if the distances are to be reduced to a horizontal plane, then the theodolite is the appropriate instrument, and by far the most convenient.

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* That correction may be computed by Trigonometry from the Earth's diameter and the height of the observer (See Exercise 44 of this Chapter); but, in practice, it is always taken from a Table to be found in works on Navigation, and entitled "Dip of the Sea Horizon," being Table VI in this volume. When such a Table is not at hand, the dip may be found approximately by the following Rule:—*Take the square root of the height in feet for the dip in minutes.* This will be correct within half a minute, unless the height exceed 100 feet.

† The correction in this case, dependent on the distance of the shore, may also be computed by Trigonometry. But it is more readily obtained from another table entitled "Dip of the Sea at different Distances," and forming Table VII of this Treatise.

Exercise 1. Suppose that a ladder cannot be placed with safety within 15 degrees of an upright position: what height can be reached by a ladder 50 feet in length?

Ans. 48 ft. $3\frac{1}{2}$ in.

2. What is the angular inclination of a regular ascent, or *gradient* as it is called, of 1 in 85; that is, of 1 foot of vertical rise for 85 of horizontal distance?

Ans. 40′ nearly.

3. If the inclination of a gradient is 1 degree, how many feet of horizontal distance does that give to a rise of 1 vertically, and how many feet of rise to a mile of horizontal distance?

Ans. The rise is 1 in 57·29, or 92·16 feet in a mile.

4. The town of B lies 17 miles East of A, and A is 19 miles North of C: what is the bearing of B from C?

Ans. N. 41° 49′ E.

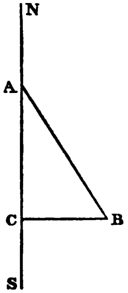
5. My writing-desk, when open, is 5·4 inches high at the back, and 1·7 inch at the front, and 20·6 inches wide from front to back: what is the angle at which it slopes?

Ans. 10° 11′.

6. A ship sails SE by S, 81 miles: what is her departure, and her difference of latitude?

Ans. Departure, 45·00 miles. Dif. of Lat., 67·35 miles.

Note. The preceding and the three following questions in Navigation are intended to be resolved by the method of *Plane Sailing*, as it is called, which proceeds on the assumption that the small part of the Earth's surface taken in is a plane. The *Departure*, means the distance that the vessel deviates from the meridian which she leaves,—that is, the number of miles she gets further East or West. Her *Difference of Latitude*, is the number of miles she gets further North or South. Thus, let AB be the distance sailed, and NS a meridian line through the ship's first position, A,—that is, a line drawn due North and South; and let BC be perpendicular to NS: then AC is called the "difference of latitude," and BC the "departure."



The angle BAC is indicated, in this question, by the given direction, SE by S. For this the *Mariner's Compass* must be referred to, which will be found delineated in Table v. Its circumference is divided into 32 equal parts, distinguishing the 32 *Points of the Compass*, as they are called,—8 in each quadrant: each of the 32 parts is also divided into 4 quarters. The number of degrees between two adjacent points must, consequently, be the eighth part of 90°, or 11° 15′. The four points, N, S, E, and W, are called the *Cardinal Points*. The direction in which a ship sails, called its *Course*, or the Bearing of one point from another, is indicated either by naming the point of the compass, as NE, or SSW, or WNW$\frac{3}{4}$W, or by naming the number of degrees from the meridian, counted either from the North or from the South towards the East or West, as N 45° E, or S 22$\frac{1}{2}$° W, or N 75° 56′ 15″ W. Any given point of the compass is expressed in degrees, by counting the number of points from the North or South, and multiplying 11° 15′ by that number, or by taking the degrees at once from the middle column in Table v.

7. Suppose a ship to sail SSW, until its departure is 198 miles: what is the distance sailed, and the difference of latitude?

Ans. Distance 517·4 miles. Dif. of Lat. 478·0 miles.

8. A ship sailed between North and West 255 miles, and thereby made her difference of latitude 157$\frac{1}{2}$ miles: on what course did she sail?

Ans. N 51° 51′ W, or NW $\frac{1}{2}$ W nearly.

NOTE. The course is first found in degrees, and, when required, these are converted into points by dividing by 11° 15′, or by referring to the middle column in Table v, taking the nearest number of degrees in that column and observing the point opposite in one of the other columns; the particular column to be taken being known from the question, or from our previous knowledge of the two cardinal points between which the vessel sails.

10. At a point 169 feet from a wall, and on a level with its base, the angle of elevation of its summit is observed to be 24° 26′: what is the height of the wall?

Ans. 76·78 + feet.

11. A surveyor, standing on the parapet of a bridge whose height he knew to be 105 feet, observed a boat at anchor at a short distance, directly opposite. Taking an observation, he found its angle of depression 26° 17′. What was the distance of the boat from the summit of the bridge, and its distance from the bridge measured horizontally,

regarding the height of the observer as an equivalent for that of the point observed on the boat?

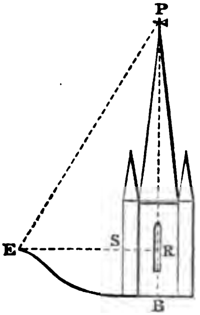
Ans. 79 and 71 yards, very nearly.

Note. When an object, such as a boat, is spoken of as a point observed, some particular point in the boat is understood; and, when there are two observations, they must both be directed to the same part of the object.

12. Wanting to determine the Sun's altitude (that is, its angular elevation), at a particular instant, and having no better instrument at hand, I placed a pole upright, 18½ feet high, and found that the shadow of its top fell on a point 23 feet 4 inches from the pole (horizontal measurement), and 37 inches higher than its base. What is the altitude computed from these data? Ans. 33° 27′.

13. At the Vernal Equinox, at noon, in Latitude 55½°, the shadow of an upright tree, growing on level ground, was 96 feet in length; and since it was a fir tree, tapering acutely towards the top, the outermost point of the shadow corresponded to the highest point of the tree. What was the tree's height? Ans. 66 feet, very nearly.

Note. At either the Vernal or the Autumnal Equinox, the Sun shines directly over the Equator, or at right angles to the Earth's axis.



16. A spire stands on a base 35 feet square. I take a station higher than the base, but opposite the middle of one side, and 20 yards distant from that side measured horizontally. From that station the elevation of the vane is observed by a common quadrant (not Hadley's) to be 56° 34′. Then, directing the sights of the quadrant horizontally, I mark the point which they strike on the upright wall, and find it 23 feet above the ground. What is the height of the vane? Ans. 140 + feet.

17. A tower stands on the brow of a uniformly sloping acclivity. I place a common quadrant at the foot of the slope and find the angular elevation of a point on the top of the perpendicular wall of the tower, to which I am

directly opposite, 41° 15′. Then, guided by a plumb line, I mark two other points on the wall directly beneath that observed at the top,—the one at the base,—the other 5 feet higher, corresponding to the height of the quadrant,—and find the elevation of the latter 18° 52′. I then suspend the plummet from the quadrant to the ground, and ascertain the distance from the plummet to the mark at the base of the wall, measured on the slope, to be 384 feet. What is the height of the wall from its base?

Ans. 199·5 – feet.

18. Placing a quadrant on a plane but not level ground, 90 feet from the base of an upright corner of a building, the distance being measured on the sloping ground, as described in Exercise 17, I take the angle of the summit of the corner, and find it 39° 27′. I then bring the telescope of the quadrant to a horizontal position, turned towards the corner, and, marking the point to which it is now directed, I find the point thus marked to be 26 feet above the base, the height of the quadrant being 5 feet. What is the height of the wall? Ans. 98·01 + feet.

19. From the brow of a steep headland a ship was observed in the roads beneath. The angle of depression of the ship, at the water line, was found to be 9° 28′; and that of the shore immediately below, in the direction of the ship, 72° 40′. What was the distance of the ship from the point of observation, and what from the shore, the perpendicular height of the precipice being known to be 254 feet, and the surface of the sea being regarded as a plane?

Ans. 1,544 and 1,444 feet, nearly.

20. Two streets meet at an acute angle. The one lies N 51° W, and the other S 48° W. The distance from the corner to a druggist's shop door in the first street is 315 yards; and the distance from the same corner to a surgeon's door in the other street is 406 yards. What is the distance in a straight line from the surgeon's door to the druggist's? Ans. 473 + yards.

21. From a vessel at anchor two rocks are observed to the westward, the one (A) bearing WNW; and the other (B), W by S, from the vessel. The rocks, being well known, are laid down on the chart, from which it is found that the former bears NNE from the latter, distant 645 yards. What are their respective distances from the vessel?

Ans. A is distant 965, and B, 1,161 yards, nearly.

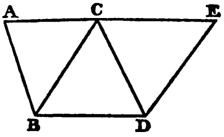
24. A castle-wall rises perpendicularly from the shore of a lake, its base being washed by the water. A rock

stands opposite, which is known to rise 50 feet above the level of the water. On the top of this rock I place my theodolite, 5 feet high, and take the angular elevation of the top of the wall, 10° 41′, and the depression of its base, 13° 25′. What is the height of the wall?

Ans. 98·50 – feet.

26. In a trigonometrical survey a base line AB is measured of 10 chains, and the following angles are taken with the theodolite :—

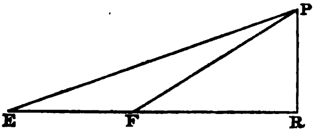
∠ABC = 48° 23′,
∠BAC = 76 13,*
∠BCD = 59 54,
∠DBC = 56 25,
∠CDE = 64 10,
∠DCE = 61 57,



The ground is every where level. DE is required.

Ans. 11·98 chains.

NOTE. On this system the great National Survey of the British Isles is conducted. A base is measured, with the utmost accuracy, as the foundation of the whole survey, and afterwards angles only are observed, triangle being added to triangle, till, after many such operations, one of the *computed* sides is also *measured* as a proof of the accuracy of all the intermediate operations.



27. In order to ascertain the height and position of a hill a flag (P) is placed on the top; and, from a station (E) on the plain below, the angle of elevation of the flag is found to be 20° 49′. A second station (F) on the plain, directly between the first station and the flag, is ascertained to be on the same level with the first and 213 feet distant from it. The elevation of the flag at the second station is observed to be 26° 12′. The distance of the flag from the nearer station, and the height of the hill above the plain, are required, the flag and the eye of

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* In practice, it is usual to measure the third angle also, of each triangle, to serve as a check on the others; but, in this place, it is better to allow the scholar to find it.

the observer being at the same height above the ground on which they stand.

Ans. Distance, 807 – ; height, 356 +, feet.

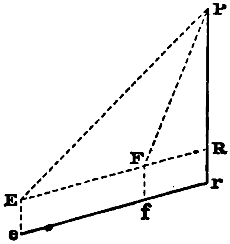
Note. When it is said that the second station is directly between the first station and the flag, it is not meant, of course, that they are all three in the same straight line, but that they are in the same vertical plane. This is determined practically by observing if a plumb line, suspended at the first station, appear to the eye to cut both the flag and the second station at once, or by using the vertical hair in the telescope instead of the plumb-line.

28. In an operation similar to the last, the angles of elevation observed at the two stations were 6° 25′ and 9° 14′; and the base (that is, the distance between the stations) was exactly 10 chains. The height of the hill is required in feet. Ans. 241 – feet.

29. In a third operation similar to the two preceding, the two observed angles of elevation were 5° 59′ and 9° 26′. The base was again 10 chains, but was found, in this instance, not to be level but inclined at an angle of 3° 4′ to the horizon, the more distant station being the higher. The height of the hill's summit above each of the stations is required. Ans. $247\frac{1}{2}$ and 283 feet, nearly.

30. The height of a fourth hill is to be determined in the same manner as the last and under similar circumstances. A base line is measured of 12 chains. The angles of elevation of the top of the hill at the two stations are 4° 23′ and 7° 40′. It is found by levelling that the ground at the station furthest from the hill is 17 feet lower than at the nearer station, and 93 feet higher than the level of a lake near it. What is the height of the top of the hill above the lake? Ans. 211 + feet.

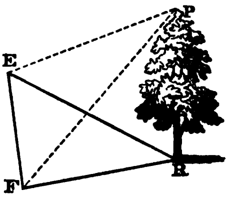
31. Desiring to know the height of the upright corner of a wall (Pr), standing on the brow of a uniformly sloping bank (er); but, having only a sextant with me, and no artificial horizon or any other means of taking an angle of elevation, I proceeded thus:—The wall being accessible, I placed a mark, R, upon it, 5 feet in height, equal to the height of my eye above the ground. I then measured 20 feet



down the bank, from r to f, and then, standing at f, I took the angle PFR, and found it 70°. I next measured 80 feet further, in the same direction, from f to e, and found the angle PER, 40°. What is the height of the corner? Ans. 103 – feet.

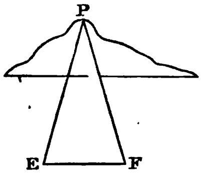
Note. In the preceding exercises, the height of an object is obtained by means of a base pointing directly towards the object, and by observing vertical angles only. But unless the object is elevated at a considerable angle (even as seen from the more remote station), or unless the ground, on which the base is measured, rises considerably as it recedes from the object, that method is not susceptible of great accuracy; for, except in the cases mentioned, the angle which the base subtends at the object (that is, the angle EPF in the two preceding diagrams) is so small, that a minute error in that angle will produce a comparatively large error in the computed distance and height of the object. It is better, therefore, to select a base fronting the object—that is, lying across the direction in which the object is seen, as in the following exercises—and to observe the horizontal and vertical angles with a theodolite, or the oblique angles with a sextant. If the observation is made at sea, or if an artificial horizon is used, the angles of elevation may also be taken with the sextant. When the theodolite is used, the base must be level, or its inclination must be ascertained. It is otherwise when the angles are taken with the sextant: the obliquity of the base is then immaterial, and entirely disregarded.

32. It is required to determine the height of a very high tree, PR. It stands in the midst of broken ground and brushwood, so that a base line cannot be measured up to the foot of the tree, or in any line directly receding from it; but there is a clear tract in the direction EF, and the top of the tree can be seen from the two points E and F, but the foot of the tree from E only: these two points are therefore taken as our stations. The base EF of 116 feet is not level; but that is of no consequence, since the angles are to be taken with the sextant. The ground at E is just so much lower than that on which the tree grows that when

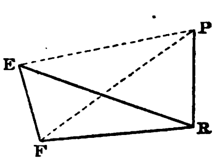


the observer stands on the former his eye is on a level with the latter. With the sextant the angles PEF and PFE are taken, and found to be 83° 20′ and 74° 45′; and the angle PER, 22° 6′. The latter is the angle of elevation at the point E, since the tree is upright and the points E and R on the same level: what is the height of the tree?* Ans. 113 feet, nearly.

33. In an experiment similar to the preceding, the base EF was level and of 2 chains in length. The angles, taken with the theodolite, were as follows: $\angle$PER $= 19°$ $46'$, $\angle$REF $= 78°\ 18'$, $\angle$RFE $= 79°\ 56'$. What was the height of the tree in this instance? Ans. 126 feet.



34. In order to ascertain the height and distance of a mountain top, P, a base, EF, of 34 chains, was measured, on a level plain, across the direction in which the mountain was seen, and the angle of elevation at E was taken and found to be 3° 18′.† When the instrument was placed at E, and the telescope directed to P, the index of the horizontal circle pointed to 182° 34′; and when the telescope was turned round till it pointed to F, the index then marked 272° 14′. The difference of these two numbers, viz. 89° 40′, gives the horizontal angle at E. In like manner the horizontal angle at F was found to be 87° 33′. To explain this, let us refer to the annexed diagram.‡ Let PR be a perpendicular dropt on the same horizontal plane with E and F; and let ER and FR be joined: ERP and FRP are both right angles; and FER and EFR are the horizontal angles taken by the theodolite at E and F, while PER is the angle of elevation at E. The height of P above the



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* The example of a tree is taken, not as the best, but that the learner may distinguish more readily, on the diagram, the vertical from the horizontal and oblique lines. The exercises which follow should be illustrated with models.

† It is usual and proper to take the angles of elevation both at E and at F, the one serving as a check upon the other, since the height may be found by either; but both are not necessary.

‡ This diagram is intended to represent the same thing as the preceding, though in a different way. But both may fail in conveying the proper ideas to the learner. He may not easily see that the angles ERP and FRP can both be right angles. The difficulty arises from our being obliged to delineate both horizontal and vertical lines on the same sheet of paper. But if the teacher will show him the same lines in a model—that is, with the point P raised above the plane of E and F, the whole will be clear to him. The only construction required for this purpose will be a straight wire placed upright on a board (to

telescope of the theodolite at E and F is required, in feet, taking the surface of the earth as a plane (that is, the line PR); and also the horizontal distance of the mountain top from the station E, in miles (that is, the line ER).

Ans. ER = 8·744 miles, and PR = 2,662 feet, nearly.

36. With a view to determine the distance and height of a rock seen rising from the sea, remote from the shore, two stations are taken along the shore, both on the high water mark. The direct distance between the two stations, being measured with a chain, is 2,527 links; and the two horizontal angles taken at the two stations (as described in Ex. 34), are 89° 15′ and 86° 21′. The angle of elevation of the highest point of the rock, taken at the first station, being 1° 48′. What is the distance of the top of the rock from the first station, in miles, and its height, in feet, above the high water level, taking 5 feet as the height of the instrument, and regarding the earth as a plane?

Ans. Distance, 4·11 + miles; height, 687 − feet.

41. A meteorologist was desirous of ascertaining, as nearly as he could, the general height of the clouds on a particular occasion. As the wind blew gently and steadily, he first obtained their velocity from that of their shadows, and found that they travelled at about the rate of 15 miles per hour. He then marked the exact time at which the margin of a cloud was right over head; and precisely 8 minutes after, observed that its angle of elevation was about 31°. What conclusion could he draw from that observation, no allowance being made for the Earth's curvature, which would make no difference worth estimating? Ans. The height of the cloud was about $1\frac{1}{5}$ mile.

43. On another occasion the height of the clouds was tried by a different method, their velocity not being regarded as uniform. A station was taken on an eminence overlooking an extensive plain. The observer waited till he perceived a small cloud, or rather part of a cloud, vertically over the station. He watched the same cloud, keeping the telescope of the theodolite directed to it, till its shadow touched a suitable point on the plain, a friend keeping his eye on the shadow while he observed the cloud. He then took its angular elevation, which was 65°. He still kept the telescope upon it till its shadow touched

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represent PR), with the horizontal lines drawn on the surface of the board, and with threads for the oblique lines; or simply a piece of wood cut into the form EPRF. (The latter will be found in the set of Models prepared for the illustration of this work, and sold by the publishers, Messrs Sutherland and Knox.)

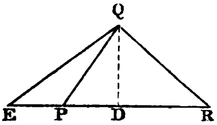
another convenient point on the plain, and then found its elevation 42°. He next measured the distance between the two points on the plain, which proved to be 923 yards. What was the height of the cloud above the station?

Ans. 1,433 yards, or ·814 of a mile.

44. Taking the Earth as a sphere of 7,912 miles in diameter, what will be the dip of the sea-horizon as seen from a mountain 3 miles in height, making no allowance for terrestrial refraction? Ans. 2° 14′.

Note. By the dip of the sea-horizon is meant the angle of depression at which the extreme surface of the sea appears below a horizontal line or plane at the point of observation.

51. The distances of three stations, P, Q, R, from each other, are as follows: PQ = 4, QR = 5, PR = 6, chains. A point E is in the same line with PR and at such a distance from P that the angle QER is of 36°. What is the distance of E from P, Q, and R?



Ans. 2·30 +, 5·63 −, and 8·30 +, chains.

CHAPTER X.

UNRESOLVED EXERCISES.

Exercises in Chapter II, Problem VI.

1. What are the logarithms of the numbers 26,583; 8,425·9; 164·51; and 98·506?

2. Of 4·387,28; 6·3̇; 305·472; and 9,845·54?

Exercises in Chapter II, Problem IX.

1. Find the natural numbers corresponding to the logarithms 2·000000; 3·220108; 5·831166; and 1·597146.

2. Of 4·820136; and 2·271377.

3. Of 1·371070; 2·472440; 0·837958; and 3·450100.

EXERCISES IN CHAPTER II, PROBLEM X.

Find the following products by logarithms :—

1. 7 × 6 × 5.
2. 258·7 × 6·844
3. 5·48 × 6·72 × 3·81 × 2·99.
4. 825 × 351 × 26·2 × 3·08, completing with ciphers.
5. 208·91 × 34·228. (See Example 2.)
6. $5{\cdot}\dot{3} \times 2{\cdot}\dot{8}$.
7. 48·574 × 5·8347 × 12·1499.
8. 786·6 × ·0428. (See Note 1.)
9. ·004,99 × ·658.
10. $67{\cdot}433 \times 5{\cdot}8628 \times {\cdot}\dot{3}\dot{8} \times {\cdot}006$.

EXERCISES IN CHAPTER II, PROBLEM XI.

1. Divide 3,312 by 92.
2. Divide 288 by 6·4.
3. Divide 92·61 by 44·1.
4. Divide 909·23 by 7·4888.
5. Divide 16·56 by 46. (See Note 1.)
6. Divide 6·8544 by 329·89.
7. Divide 4·83 by 0.882. (See Note 2.)
8. Divide 0·6076 by ·0084.
9. Divide 0·882 by 4·833. (See Note 3.)
10. Divide .0084 by 0·6076.

EXERCISES IN CHAPTER II, PROBLEM XII.

Compute the fourth term of each of the following proportions, by logarithms :—

1. As 279 : 651 :: 657 : ——.
2. As 94·25 : 486·3 :: 8·828 : ——.
3. As 3·2496 : 54·853 :: 15 : ——.
4. As 0·28 : 6·75 :: 4·838 : ——. (See Note 2.)
5. As ·0656 : ·0835 :: 2·24 : ——.
6. As ·00958 : ·06534 :: ·02755 : ——.
7. As 25 : 0·669 :: 48 : ——. (See Note 3.)
8. As 94·3 : 94·2 :: ·08886 : ——.
9. As 0·628 : 0·723 :: ·05 : ——.
10. As 2 : 0·66 :: ·048 : ——. (See Note 4.)
11. As 0·484,66 : ·058,32 :: ·079,63 : ——.
12. As 892 : 24·4 :: 8·52 : ——. (See Note 5.)
13. As 73·598 : 0·694 :: ·058 : ——.

EXERCISES IN CHAPTER II, PROBLEM XIII.

1. Square the numbers 25 ; 38·3 ; 5·4962 ; and $48{\cdot}3\dot{8}$.

2. Cube the numbers 15 ; $6{\cdot}\dot{3}$; and 18·499.

3. Square 384·5, and cube 198, completing the results with ciphers.

4. Square 0·852,63 ; ·0179 ; and ${\cdot}00\dot{8}$. (See the Note.)

5. Cube 0·2835 and ·0899.

EXERCISES IN CHAPTER II, PROBLEM XV.

1. Extract the square roots of 84,100 ; 88·36 ; 285,168 ; and 10.

2. Extract the cube roots of 6,859 ; 3,375,000 ; 857·4 ; and 600.

3. Find the square roots of ·0081 ; 0·658 ; ·00055 ; and $0{\cdot}\dot{4}$. (See Note.)

4. Calculate the cube roots of ·000,585 ; 0·5 ; ·0688 ; and $0{\cdot}\dot{8}$.

EXERCISES IN CHAPTER III.

1. An ancient Roman mile was equivalent to 1612 yards of British Imperial Measure. Convert 556 Roman miles into Imperial miles.

2. If 8.546 acres of land are valued at £564, 10s., what should be the value of 2·868 acres, at the same rate?

3. What is the interest of £457, 8s., for 7 months, at 4¼ per cent. per annum?

4. Compute the interest of £64 : 10 : 9, for 288 days at 3¾ per cent. per annum.

5. Calculate the amount of £10, for fifty years, at 4 per cent. per annum, compound interest.

6. What sum will one penny amount to in 500 years, if accumulating, all the while, at 5 per cent. per annum, compound interest?

7. What is the compound interest of £383 for 5½ years at 3½ per cent. per annum?

8. Find the diameter of a circle, the circumference of which is 408·2.

9. What is the area, in acres, of a circle whose radius is 2·85 chains?

10. Determine the area of an ellipse, the transverse axis being 7·38, and the conjugate 4·71.

11. Compute the number of cubic feet in a cone, whose base is 356 inches in diameter, and whose height is 98 inches.

12. What is the weight of an ivory ball $1\frac{3}{4}$ inches in diameter, at the rate of 1820 ounces (Av.) to the cubic foot?

Exercises in Chapter VI, Problem I.

1. What are the log. sines of 24°, of 68°, of 35° 32′, and of 84° 15′?

2. What is the log. tangent of 32° 12′, and that of 48° 40′?

3. What is the log. cosine of 18° 24′, and the log. secant of 50° 8′?

Exercises in Chapter VI, Problem II.

Find the log. sine of 22° 48′ 54″, the log. tangent of 63° 25′ 16″, the log. secant of 38° 14′ 5″, and the log. cosine of 76° 0′ 48″.

Exercises in Chapter VI, Problem V.

What are the log. sines of 104° 18′, 126° 12′ 24″, and 164° 54′ 48″?

Exercises in Chapter VI, Problem VII.

What angles correspond to log. sine 9·520271, log. tangent 10·407574, and log. secant 10·566325?

2. Find the numbers of degrees and minutes corresponding to log. sine 9·284776, log. tan 10·550548, and log. secant 11·132496.

3. What are the angles answering to log. cosine 9·995555, to log. cotan 10·508800, and to log. cosecant 10·030568?

Exercises in Chapter VII, Problem I.

1. The acute angle at the base of a right-angled triangle is $18\frac{3}{4}$°: what is that at the vertex?

2. The acute angle at the vertex being 84° 13½′, what is that at the base?

3. If the one acute angle contain 28° 44′ 55″, how many degrees are in the other?

EXERCISES IN CHAPTER VII, PROBLEM II.

1. Hyp. = 5438. Acute angle at base = 35°. Perp. required.

2. Hyp. = 2·49. Acute angle at vertex = 43° 25′. Base required.

3. Hyp. = 1000. One acute angle = 36° 16′. What are the legs?

4. Hyp. = ·04. Angle at base = 43° 40′. Sides required.

5. Hyp. = 8·2958. Acute angle at vertex = 54° 23′. Perp. required.

EXERCISES IN CHAPTER VII, PROBLEM III.

1. Base = 845. Adjacent acute angle = 28° 25′. Hyp. required.

2. Perp. = 18·48. Vertical angle = 59° 12′. Hyp. required.

3. Base = 4·843. Vertical angle = 7°. Hyp. required.

4. Compute, by Trigonometry, the diagonal of a square whose side is 749.

5. Perp. = 583·26. Acute angle at base = 18° 48¾′. Hyp. required.

EXERCISES IN CHAPTER VII, PROBLEM IV.

1. Base = 14·04. Acute angle adjacent = 72° 48′. Perp. required.

2. Perp. = 3 ft. 4 in. Vertical angle = 22° 28′. Base required.

3. Base = 384. Vertical angle = 48° 26′ 12″. Perp. required.

4. Base = 0·440,83. Acute angle at the base = 33° 38·2′. Perp. required.

EXERCISES IN CHAPTER VII, PROBLEM V.

1. Hyp. = 12¾. Perp. = 8¼. The angles are required.
2. Base = $0{\cdot}\dot{8}$. Perp. = $1{\cdot}\dot{2}$. What are the angles?
3. The base of an isosceles triangle is 5, and each of its equal sides double the base. What is the vertical angle?

EXERCISES IN CHAPTER VII, PROBLEM VI.

1. The legs being 2 and 3, what are the acute angles?
2. Base = 3 yds. 1 ft. 6 in. Perp. = 5 yds. 9 in. What are the acute angles?
3. What is the angle of inclination of a gradient rising at the rate of 1 in 28?
4. The legs being ·0096 and ·0192, what are the acute angles?

EXERCISES IN CHAPTER VII, PROBLEM VII.

1. Hyp. = 8 ft. 5¾ in. Base = 6 ft. 4¼ in. Perp. required.
2. Hyp. = 8125. Perp. = 3125. Base required.
3. Hyp. = 0·13167. Base = ·02660. Perp. required.

EXERCISES IN CHAPTER VII, PROBLEM VIII.

1. Base = 12·6. Perp. = 1·33. Hyp. required.
2. Base = 26. Perp. = 128·7. Hyp. required.
3. Legs ·0784 and ·0377. Hyp. required.

PROMISCUOUS EXERCISES IN RIGHT-ANGLED TRIANGLES.

1. Hyp. = 101. Perp. = 99. Base required.
2. One of the equal sides of an isosceles triangle is 19·49, and the angle at the vertex 45°. What is the base?
3. Compute, by Trigonometry, the side of an equilateral triangle whose altitude is 100.
4. The three sides of a triangle are 193, 193, and 190. Taking the shortest side as the base, what is the altitude, and what are the angles at the base?
5. The largest of the pyramids of Egypt, being more correctly a frustum of a pyramid, is 480 feet high, and stands on a base 745 feet square, its summit being 32 feet

square. At what angle are the sides inclined to the horizon?

6. One of the acute angles of a right-angled triangle is four times the other. The shorter leg is 10. What is the longer leg, and the hypothenuse?

EXERCISES IN CHAPTER VIII, PROBLEM I.

1. Given, the angles at the base $33\frac{3}{4}^\circ$ and $59\frac{1}{2}^\circ$. Required the angle at the vertex.

2. $\angle A=84^\circ\ 49'$. $\angle B=63^\circ\ 28'$. $\angle C$ is required.

3. $\angle B=49^\circ\ 10'\ 53''$. $\angle C=72^\circ\ 57'\ 15''$. $\angle A$ is required.

4. The sum of the angles at the base of a triangle is $114^\circ\ 48\frac{3}{4}'$. What is the angle at the vertex?

5. The angle at the vertex being $77\frac{1}{4}^\circ$, what is the sum of the angles at the base?

EXERCISES IN CHAPTER VIII, PROBLEM II.

1. Given, $\angle A=49^\circ\ 54'$, $\angle B=72^\circ\ 28'$, side $a=769$. Required side b.

2. The three angles of a triangle are respectively 54°, 69°, and 57°. The shortest side is 8·58 chains. What are the other two sides?

3. Two angles of a triangle are severally $39^\circ\ 10'$ and $104^\circ\ 48'$. The side opposite the former is 48 yds. 2 ft. What is the side opposite the latter?

4. What is the third side of the last-mentioned triangle?

EXERCISES IN CHAPTER VIII, PROBLEM III.

1. Side $a=4{\cdot}94$. Side $b=5{\cdot}09$. $\angle B=118^\circ$. $\angle A$ is required.

2. Sides a and $b=5$ and 6. $\angle A=54\frac{1}{2}^\circ$. $\angle B$ is required, which is acute.

3. Sides a and $b=0{\cdot}3$ and $0{\cdot}4$. $\angle A=38^\circ\ 43'$. $\angle B$ is required, which is obtuse.

4. Side $a=548{\cdot}3$. Side $b=499{\cdot}4$. $\angle A=76^\circ\ 23'$. What is $\angle B$?

5. Side $a=10$. Side $b=11$. $\angle A=57^\circ\ 4'$. What is $\angle B$?

EXERCISES IN CHAPTER VIII, PROBLEM IV.

Compute the third side in each of the five triangles described in the Exercises under the preceding Problem.

EXERCISES IN CHAPTER VIII, PROBLEM V.

1. Sides, 4 and 5. Included angle, 66°. What are the other angles?
2. Sides, 8¾ and 9⅝. Vertical angle, 18½°. The other angles are required.
3. Sides, 2⅔ and 2⅓. Included angle, 96° 20′. Compute the other angles.

EXERCISE IN CHAPTER VIII, PROBLEM VI.

Calculate the third side of each of the three triangles described in the preceding Exercises, under Problem V.

EXERCISES IN CHAPTER VIII, PROBLEM VII.

1. The three sides being 1636, 1524, and 1540, what are the three angles?
2. Sides, 11·46, 9·68, and 8·674. What are the angles?

EXERCISES IN CHAPTER IX.

NOTE. Instead of giving a new set of questions in this place, it has been thought better merely to give repetitions of those in Chapter IX, with the figures altered. Thus, of the following questions, the first is understood to be the same as Exercise 1 of that Chapter, substituting 12 degrees for 15, and 36 feet for 50.

1. ....12 degrees...............36 feet.........?
2. .... 1 in 100; ....?
3. ....1½ degree,...............?
4. ....18 miles West......20 miles South......?
5. ....5½......... 2¼........20½.........?
6. ....NNW, 12 miles.............?

7. .... E by N, 86 miles............?

8. .... between S and W, 200 miles,.......84¼ miles: ....?

10. ....315............10° 38′: ....?

11. .... 87 feet, ...19° 59′.............?

12. .... 12 feet......25 feet......30 inches lower......?

13. ....Latitude 51° 25′............75 feet ....?

16. ....24 feet square.......36 yards...... 33° 39′...... 16½ feet.........?

17. ....28° 38′.......5 feet ....12° 12′.......496 feet. ....?

18. ......101 feet......34° 45′......20 feet above the base ....?

19. ....8° 14′;.........78° 22′ ....168 feet,......?

20. ......N 22° E, ...N 28 W......300......200......?

21. ....ESE;........E by N,.......SW by S........500 yards..........?

24. ....108.........angular depression of the top...... 2° 49′, and of the base 26° 52′ ....?

26. ....1 mile,......... :—

∠ABC = 54° 16′.
∠BAC = 73° 28′.
∠BCD = 61° 33′.
∠DBC = 64° 20′.
∠CDE = 58° 59′.
∠DCE = 56° 11′.

DE is required in feet.

27. ....15° 31′.........300 feet.........21° 54′.........

29. ......6° 3′ and 8° 30′......8 chains......0° 44′......, the more distant station being the lower. ....

30. ....8 chains.......3° 50′ and 7° 29′.......26 feet higher......12 feet higher......?

31. ......5 feet......1 chain......38° 10′......1 chain...... 21° 42′......?

32. ....The base EF of 100 feet.........86° 18′ and 70° 47′; and the angle PER, 15° 42′......?

34. ....a base of 30 chains.......angle of elevation at E = 2° 53′. Horizontal angle at E = 86°. Horizontal angle at F = 91° 13′.........

36. ....12·88 chains;............91° 36′ and 84° 2′. ....2° 12′.

41. ....36 miles per hour ....5 minutes after ....24° 42′. What was the height of the cloud in feet?

43. ....71°.........45°.........657 yards. What was the height of the cloud in feet?

44. ....2 miles............?

51. ....PQ = 2, QR = 3, PR = 4.........45°........?

TABLES.

Table I.—*Logarithms of Numbers from* 1 *to* 250, *with their Indices.*

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0·000000	51	1·707570	101	2·004321	151	2·178977	201	2·303196
2	0·301030	52	1·716003	102	2·008600	152	2·181844	202	2·305351
3	0·477121	53	1·724276	103	2·012837	153	2·184691	203	2·307496
4	0·602060	54	1·732394	104	2.017033	154	2·187521	204	2·309630
5	0·698970	55	1·740363	105	2.021189	155	2·190332	205	2·311754
6	0·778151	56	1·748188	106	2·025306	156	2·193125	206	2·313867
7	0·845098	57	1·755875	107	2·029384	157	2·195900	207	2·315970
8	0·903090	58	1·763428	108	2·033424	158	2·198657	208	2·318063
9	0·954243	59	1·770852	109	2·037426	159	2·201397	209	2·320146
10	1·000000	60	1·778151	110	2·041393	160	2·204120	210	2·322219
11	1·041393	61	1·785330	111	2·045323	161	2·206826	211	2·324282
12	1·079181	62	1·792392	112	2·049218	162	2·209515	212	2·326336
13	1·113943	63	1·799341	113	2·053078	163	2·212188	213	2·328380
14	1·146128	64	1·806180	114	2·056905	164	2·214844	214	2·330414
15	1·176091	65	1·812913	115	2·060698	165	2·217484	215	2·332438
16	1·204120	66	1·819544	116	2·064458	166	2·220108	216	2·334454
17	1·230449	67	1·826075	117	2·068186	167	2·222716	217	2·336460
18	1·255273	68	1·832509	118	2·071882	168	2·225309	218	2·338456
19	1·278754	69	1·838849	119	2·075547	169	2·227887	219	2·340444
20	1·301030	70	1·845098	120	2·079181	170	2·230449	220	2·342423
21	1·322219	71	1·851258	121	2·082785	171	2·232996	221	2·344392
22	1·342423	72	1·857332	122	2·086360	172	2·235528	222	2·346353
23	1·361728	73	1·863323	123	2·089905	173	2·238046	223	2·348305
24	1·380211	74	1·869232	124	2·093422	174	2·240549	224	2·350248
25	1·397940	75	1·875061	125	2·096910	175	2·243038	225	2·352183
26	1·414973	76	1·880814	126	2·100371	176	2·245513	226	2·354108
27	1·431364	77	1·886491	127	2·103804	177	2·247973	227	2·356026
28	1·447158	78	1·892095	128	2·107210	178	2·250420	228	2·357935
29	1·462398	79	1·897627	129	2·110590	179	2·252853	229	2·359835
30	1·477121	80	1·903090	130	2·113943	180	2·255273	230	2·361728
31	1·491362	81	1·908485	131	2·117271	181	2·257679	231	2·363612
32	1·505150	82	1·913814	132	2·120574	182	2·260071	232	2·365488
33	1·518514	83	1·919078	133	2·123852	183	2·262451	233	2·367356
34	1·531479	84	1·924279	134	2·127105	184	2·264818	234	2·369216
35	1·544068	85	1·929419	135	2·130334	185	2·267172	235	2·371068
36	1·556303	86	1·934498	136	2·133539	186	2·269513	236	2·372912
37	1·568202	87	1·939519	137	2·136721	187	2·271842	237	2·374748
38	1·579784	88	1·944483	138	2·139879	188	2·274158	238	2·376577
39	1·591065	89	1·949390	139	2·143015	189	2·276462	239	2·378398
40	1·602060	90	1·954243	140	2·146128	190	2·278754	240	2·380211
41	1·612784	91	1·959041	141	2·149219	191	2·281033	241	2·382017
42	1·623249	92	1·963788	142	2·152288	192	2·283301	242	2·383815
43	1·633468	93	1·968483	143	2·155336	193	2·285557	243	2·385606
44	1·643453	94	1·973128	144	2·158362	194	2·287802	244	2·387390
45	1·653213	95	1·977724	145	2·161368	195	2·290035	245	2·389166
46	1·662758	96	1·982271	146	2·164353	196	2·292256	246	2·390935
47	1·672098	97	1·986772	147	2·167317	197	2·294466	247	2·392697
48	1·681241	98	1·991226	148	2·170262	198	2·296665	248	2·394452
49	1·690196	99	1·995635	149	2·173186	199	2·298853	249	2·396199
50	1·698970	100	2·000000	150	2·176091	200	2·301030	250	2·397940

N.	0	1	2	3	4	5	6	7	8	9
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174
102	8600	9026	9451	9876	0̅300	0̅724	1̅147	1̅570	1̅993	2̅415
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616
104	7033	7451	7868	8284	8700	9116	9532	9947	0̅361	0̅775
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978
107	9384	9789	0̅195	0̅600	1̅004	1̅408	1̅812	2̅216	2̅619	3̅021
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028
109	7426	7825	8223	8620	9017	9414	9811	0̅207	0̅602	0̅998
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830
112	9218	9606	9993	0̅380	0̅766	1̅153	1̅538	1̅924	2̅309	2̅694
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	0̅320
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815
117	8186	8557	8928	9298	9668	0̅038	0̅407	0̅776	1̅145	1̅514
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819
120	9181	9543	9904	0̅266	0̅626	0̅987	1̅347	1̅707	2̅067	2̅426
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552
123	9905	0̅258	0̅611	0̅963	1̅315	1̅667	2̅018	2̅370	2̅721	3̅071
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	0̅026
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	0̅253
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	0̅245
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	0̅012
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564
138	9879	0̅194	0̅508	0̅822	1̅136	1̅450	1̅763	2̅076	2̅389	2̅702
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911
141	9219	9527	9835	0̅142	0̅449	0̅756	1̅063	1̅370	1̅676	1̅982
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061
144	8362	8664	8965	9266	9567	9868	0̅168	0̅469	0̅769	1̅068
145	161368	1667	1967	2266	2564	2863	3161	3460	3758	4055
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802

N.	0	1	2	3	4	5	6	7	8	9
150	176091	6381	6670	6959	7248	7536	7825	8113	8401	8689
151	8977	9264	9552	9839	0̄126	0̄413	0̄699	0̄986	1̄272	1̄558
152	181844	2129	2415	2700	2985	3270	3555	3839	4123	4407
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	0̄051
155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382
158	8657	8932	9206	9481	9755	0̄029	0̄303	0̄577	0̄850	1̄124
159	201397	1670	1943	2216	2488	2761	3033	3305	3577	3848
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247
162	9515	9783	0̄051	0̄319	0̄586	0̄853	1̄121	1̄388	1̄654	1̄921
163	212188	2454	2720	2986	3252	3518	3783	4049	4314	4579
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846
166	220108	0370	0631	0892	1153	1414	1675	1936	2196	2456
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630
169	7887	8144	8400	8657	8913	9170	9426	9682	9938	0̄193
170	230449	0704	0960	1215	1470	1724	1979	2234	2488	2742
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795
173	8046	8297	8548	8799	9049	9299	9550	9800	0̄050	0̄300
174	240549	0799	1048	1297	1546	1795	2044	2293	2541	2790
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266
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177	7973	8219	8464	8709	8954	9198	9443	9687	9932	0̄176
178	250420	0664	0908	1151	1395	1638	1881	2125	2368	2610
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833
182	260071	0310	0548	0787	1025	1263	1501	1739	1976	2214
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279
186	9513	9746	9980	0̄213	0̄446	0̄679	0̄912	1̄144	1̄377	1̄609
187	271842	2074	2306	2538	2770	3001	3233	3464	3696	3927
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525
190	8754	8982	9211	9439	9667	9895	0̄123	0̄351	0̄578	0̄806
191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812
195	290035	0257	0480	0702	0925	1147	1369	1591	1813	2034
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635
199	8853	9071	9289	9507	9725	9943	0̄161	0̄378	0̄595	0̄813

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202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417
204	9630	9843	0̄056	0̄268	0̄481	0̄693	0̄906	1̄118	1̄330	1̄542
205	311754	1966	2177	2389	2600	2812	3023	3234	3445	3656
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938
209	320146	0354	0562	0769	0977	1184	1391	1598	1805	2012
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176
213	8380	8583	8787	8991	9194	9398	9601	9805	0̄008	0̄211
214	330414	0617	0819	1022	1225	1427	1630	1832	2034	2236
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257
218	8456	8656	8855	9054	9253	9451	9650	9849	0̄047	0̄246
219	340444	0642	0841	1039	1237	1435	1632	1830	2028	2225
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	0̄054
224	350248	0442	0636	0829	1023	1216	1410	1603	1796	1989
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646
229	9835	0̄025	0̄215	0̄404	0̄593	0̄783	0̄972	1̄161	1̄350	1̄539
230	361728	1917	2105	2294	2482	2671	2850	3048	3236	3424
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030
234	9216	9401	9587	9772	9958	0̄143	0̄328	0̄513	0̄698	0̄883
235	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216
239	8398	8580	8761	8943	9124	9306	9487	9668	9849	0̄030
240	380211	0392	0573	0754	0934	1115	1296	1476	1656	1837
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212
244	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989
245	9166	9343	9520	9698	9875	0̄051	0̄228	0̄405	0̄582	0̄759
246	390935	1112	1288	1464	1641	1817	1993	2169	2345	2521
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025
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252	401401	1573	1745	1917	2089	2261	2433	2605	2777	2949
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764
257	9933	0̄102	0̄271	0̄440	0̄609	0̄777	0̄946	1̄114	1̄283	1̄451
258	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791
263	9956	0̄121	0̄286	0̄451	0̄616	0̄781	0̄945	1̄110	1̄275	1̄439
264	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082
265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591
269	9752	9914	0̄075	0̄236	0̄398	0̄559	0̄720	0̄881	1̄042	1̄203
270	431364	1525	1685	1846	2007	2167	2328	2488	2649	2809
271	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409
272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175
275	9333	9491	9648	9806	9964	0̄122	0̄279	0̄437	0̄594	0̄752
276	440909	1066	1224	1381	1538	1695	1852	2009	2166	2323
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449
279	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552
281	8706	8861	9015	9170	9324	9478	9633	9787	9941	0̄095
282	450249	0403	0557	0711	0865	1018	1172	1326	1479	1633
283	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165
284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214
286	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731
287	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242
288	9392	9543	9694	9845	9995	0̄146	0̄296	0̄447	0̄597	0̄748
289	460898	1048	1198	1348	1499	1649	1799	1948	2098	2248
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744
291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675
295	9822	9969	0̄116	0̄263	0̄410	0̄557	0̄704	0̄851	0̄998	1̄145
296	471292	1438	1585	1732	1878	2025	2171	2318	2464	2610
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526
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302	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157
305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410
308	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818
309	9958	0̄099	0̄239	0̄380	0̄520	0̄661	0̄801	0̄941	1̄081	1̄222
310	491362	1502	1642	1782	1922	2062	2201	2341	2481	2621
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406
313	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791
314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550
316	9687	9824	9962	0̄099	0̄236	0̄374	0̄511	0̄648	0̄785	0̄922
317	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291
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319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370
321	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721
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323	9203	9337	9471	9606	9740	9874	0̄009	0̄143	0̄277	0̄411
324	510545	0679	0813	0947	1081	1215	1349	1482	1616	1750
325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415
327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697
331	9828	9959	0̄090	0̄221	0̄353	0̄484	0̄615	0̄745	0̄876	1̄007
332	521138	1269	1400	1530	1661	1792	1922	2053	2183	2314
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788
338	8917	9045	9174	9302	9430	9559	9687	9815	9943	0̄072
339	530200	0328	0456	0584	0712	0840	0968	1096	1223	1351
340	1479	1607	1734	1862	1990	2117	2245	2372	2500	2627
341	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167
343	5294	5421	5547	5674	5800	5927	6053	6179	6306	6432
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951
346	9076	9202	9327	9452	9578	9703	9829	9954	0̄079	0̄204
347	540329	0455	0580	0705	0830	0955	1080	1205	1330	1454
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944

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351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652
353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	0̄106
355	550228	0351	0473	0595	0717	0840	0962	1084	1206	1328
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547
357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387
361	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589
362	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787
363	9907	0̄026	0̄146	0̄265	0̄385	0̄504	0̄624	0̄743	0̄863	0̄982
364	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257
371	9374	9491	9608	9725	9842	9959	0̄076	0̄193	0̄309	0̄426
372	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755
374	2872	2988	8104	3220	3336	3452	3568	3684	3800	3915
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377
378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669
380	9784	9898	0̄012	0̄126	0̄241	0̄355	0̄469	0̄583	0̄697	0̄811
381	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838
389	9950	0̄061	0̄173	0̄284	0̄396	0̄507	0̄619	0̄730	0̄842	0̄953
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774
398	9883	9992	0̄101	0̄210	0̄319	0̄428	0̄537	0̄646	0̄755	0̄864
399	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951

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400	602060	2169	2277	2386	2494	2603	2711	2819	2928	3036
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488
407	9594	9701	9808	9914	$\bar{0}$021	$\bar{0}$128	$\bar{0}$234	$\bar{0}$341	$\bar{0}$447	$\bar{0}$554
408	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895
414	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943
415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989
416	9093	9198	9302	9406	9511	9615	9719	9824	9928	$\bar{0}$032
417	620136	0240	0344	0448	0552	0656	0760	0864	0968	1072
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179
421	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263
424	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308
426	9410	9512	9613	9715	9817	9919	$\bar{0}$021	$\bar{0}$123	$\bar{0}$224	$\bar{0}$326
427	630428	0530	0631	0733	0835	0936	1038	1139	1241	1342
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376
431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383
432	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388
433	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390
434	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387
436	9486	9586	9686	9785	9885	9984	$\bar{0}$084	$\bar{0}$183	$\bar{0}$283	$\bar{0}$382
437	640481	0581	0680	0779	0879	0978	1077	1177	1276	1375
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366
439	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340
441	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324
442	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306
443	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285
444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237
446	9335	9432	9530	9627	9724	9821	9919	$\bar{0}$016	$\bar{0}$113	$\bar{0}$210
447	650308	0405	0502	0599	0696	0793	0890	0987	1084	1181
448	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150
449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116

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451	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042
452	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002
453	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960
454	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870
456	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821
457	9916	0̄011	0̄106	0̄201	0̄296	0̄391	0̄486	0̄581	0̄676	0̄771
458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718
459	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663
460	2758	2852	2947	3041	3135	3230	3324	3418	3512	3607
461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548
462	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487
463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424
464	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293
466	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224
467	9317	9410	9503	9596	9689	9782	9875	9967	0̄060	0̄153
468	670246	0339	0431	0524	0617	0710	0802	0895	0988	1080
469	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929
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472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337
478	9428	9519	9610	9700	9791	9882	9973	0̄063	0̄154	0̄245
479	680336	0426	0517	0607	0698	0789	0879	0970	1060	1151
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055
481	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957
482	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857
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484	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547
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487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220
489	9309	9398	9486	9575	9664	9753	9841	9930	0̄019	0̄107
490	690196	0285	0373	0462	0550	0639	0728	0816	0905	0993
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759
493	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639
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495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394
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497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883

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501	9838	9924	0̄011	0̄098	0̄184	0̄271	0̄358	0̄444	0̄531	0̄617
502	700704	0790	0877	0963	1050	1136	1222	1309	1395	1482
503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205
505	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065
506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336
511	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185
512	9270	9355	9440	9524	9609	9694	9779	9863	9948	0̄033
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514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407
517	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084
519	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754
521	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587
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523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	0̄077
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526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830
532	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646
533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893
537	9974	0̄055	0̄136	0̄217	0̄298	0̄378	0̄459	0̄540	0̄621	0̄702
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508
539	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117
541	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919
542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720
543	4800	4880	4960	5040	5120	5199	5279	5359	5439	5519
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317
545	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113
546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493
549	9572	9651	9731	9810	9889	9968	0̄047	0̄126	0̄205	0̄284

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552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659
562	9736	9814	9891	9968	0̄045	0̄123	0̄200	0̄277	0̄354	0̄431
563	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506
567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799
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571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320
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573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592
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576	760422	0498	0573	0649	0724	0799	0875	0950	1025	1101
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604
579	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101
581	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848
582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338
584	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082
585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	0̄042
589	770115	0189	0263	0336	0410	0484	0557	0631	0705	0778
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354
599	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079

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600	778151	8224	8296	8368	8441	8513	8585	8658	8730	8802
601	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524
602	9596	9669	9741	9813	9885	9957	$\overline{0}$029	$\overline{0}$101	$\overline{0}$173	$\overline{0}$245
603	780317	0389	0461	0533	0605	0677	0749	0821	0893	0965
604	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510
616	9581	9651	9722	9792	9863	9933	$\overline{0}$004	$\overline{0}$074	$\overline{0}$144	$\overline{0}$215
617	790285	0356	0426	0496	0567	0637	0707	0778	0848	0918
618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620
619	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022
621	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721
622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418
623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890
628	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582
629	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272
630	9341	9409	9478	9547	9616	9685	9754	9823	9892	9961
631	800029	0098	0167	0236	0305	0373	0442	0511	0580	0648
632	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335
633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112
640	6180	6248	6316	6384	6451	6519	6587	6655	6723	6790
641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467
642	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143
643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492
645	9560	9627	9694	9762	9829	9896	9964	$\overline{0}$031	$\overline{0}$098	$\overline{0}$165
646	810233	0300	0367	0434	0501	0569	0636	0703	0770	0837
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178
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652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478
660	9544	9610	9676	9741	9807	9873	9939	0̄004	0̄070	0̄136
661	820201	0267	0333	0399	0464	0530	0595	0661	0727	0792
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448
663	1514	1579	1644	1710	1775	1841	1906	1972	2037	2103
664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882
676	9947	0̄011	0̄075	0̄139	0̄204	0̄268	0̄332	0̄396	0̄460	0̄525
677	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083
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682	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357
683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415
691	9478	9541	9604	9667	9729	9792	9855	9918	9981	0̄043
692	840106	0169	0232	0294	0357	0420	0482	0545	0608	0671
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922
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697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415
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703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972
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711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668
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720	7332	7393	7453	7513	7574	7634	7694	7755	7815	7875
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722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078
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725	860338	0398	0458	0518	0578	0637	0697	0757	0817	0877
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072
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732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045
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737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998
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742	870404	0462	0521	0579	0638	0696	0755	0813	0872	0930
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098
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752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889
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758	9669	9726	9784	9841	9898	9956	$\overline{0}013$	$\overline{0}070$	$\overline{0}127$	$\overline{0}185$
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761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037
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765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870
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771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561
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775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806
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778	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482
779	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595
781	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151
782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706
783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261
784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766
794	9821	9875	9930	9985	$\overline{0}039$	$\overline{0}094$	$\overline{0}149$	$\overline{0}203$	$\overline{0}258$	$\overline{0}312$
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796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492
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802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202
804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281
806	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820
807	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358
808	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895
809	7949	8002	8056	8109	8163	8217	8270	8324	8378	8431
810	8485	8539	8592	8646	8699	8753	8807	8860	8914	8967
811	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503
812	9556	9609	9663	9716	9770	9823	9877	9930	9984	0̄037
813	910091	0144	0197	0251	0304	0358	0411	0464	0518	0571
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104
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816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231
819	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290
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829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549
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832	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634
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836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776
843	5828	5879	5931	5982	6034	6085	6137	6188	6239	6291
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857
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851	9930	9981	0̄032	0̄083	0̄134	0̄185	0̄236	0̄287	0̄338	0̄389
852	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898
853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953
861	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457
862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463
864	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966
865	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468
866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470
868	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970
869	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968
871	940018	0068	0118	0168	0218	0267	0317	0367	0417	0467
872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964
873	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462
874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829
891	9878	9926	9975	0̄024	0̄073	0̄121	0̄170	0̄219	0̄267	0̄316
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194

N.	0	1	2	3	4	5	6	7	8	9
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8116
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947
912	9995	0̄042	0̄090	0̄138	0̄185	0̄233	0̄280	0̄328	0̄376	0̄423
913	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899
914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369
932	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835
933	9882	9928	9975	0̄021	0̄068	0̄114	0̄161	0̄207	0̄254	0̄300
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543
941	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005
942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678

N.	0	1	2	3	4	5	6	7	8	9
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850
977	9895	9939	9983	0̄028	0̄072	0̄117	0̄161	0̄206	0̄250	0̄294
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591
990	5635	5679	5723	5767	5811	5854	5898	5942	5986	6030
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957

0°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	Inf. Neg.	Infinite.	Inf. Neg.	Infinite.	10.000000	10 000000	60
1	6.463726	13.536274	6.463726	13.536274	000000	000000	59
2	764756	235244	764756	235244	000000	000000	58
3	940847	059153	940847	059153	000000	000000	57
4	7.065786	12.934214	7.065786	12.934214	000000	000000	56
5	7.162696	12.837304	7.162696	12.837304	10.000000	10.000000	55
6	241877	758123	241878	758122	000001	9.999999	54
7	308824	691176	308825	691175	000001	999999	53
8	366816	633184	366817	633183	000001	999999	52
9	417968	582032	417970	582030	000001	999999	51
10	7.463725	12.536275	7.463727	12.536273	10.000002	9.999998	50
11	505118	494882	505120	494880	000002	999998	49
12	542906	457094	542909	457091	000003	999997	48
13	577668	422332	577672	422328	000003	999997	47
14	609853	390147	609857	390143	000004	999996	46
15	7.639816	12.360184	7.639820	12.360180	10.000004	9.999996	45
16	667845	332156	667849	332151	000005	999995	44
17	694173	305827	694179	305821	000005	999995	43
18	718997	281003	719003	280997	000006	999994	42
19	742477	257523	742484	257516	000007	999993	41
20	7.764754	12.235246	7.764761	12.235239	10.000007	9.999993	40
21	785943	214057	785951	214049	000008	999992	39
22	806146	193854	806155	193845	000009	999991	38
23	825451	174549	825460	174540	000010	999990	37
24	843934	156066	843944	156056	000011	999989	36
25	7.861662	12.138338	7.861674	12.138326	10.000012	9.999988	35
26	878695	121305	878708	121292	000012	999988	34
27	895085	104915	895099	104901	000013	999987	33
28	910879	089121	910894	089106	000014	999986	32
29	926119	073881	926134	073866	000015	999985	31
30	7.940842	12.059158	7.940858	12.059142	10.000017	9.999983	30
31	955082	044918	955100	044900	000018	999982	29
32	968870	031130	968889	031111	000019	999981	28
33	982233	017767	982253	017747	000020	999980	27
34	995198	004802	995219	004781	000021	999979	26
35	8.007787	11.992213	8.007809	11.992191	10.000023	9.999977	25
36	020021	979979	020045	979955	000024	999976	24
37	031919	968081	031945	968055	000025	999975	23
38	043501	956499	043527	956473	000027	999973	22
39	054781	945219	054809	945191	000028	999972	21
40	8.065776	11.934224	8.065806	11.934194	10.000029	9.999971	20
41	076500	923500	076531	923469	000031	999969	19
42	086965	913035	086997	913003	000032	999968	18
43	097183	902817	097217	902783	000034	999966	17
44	107167	892833	107202	892797	000036	999964	16
45	8.116926	11.883074	8.116963	11.883037	10.000037	9.999963	15
46	126471	873529	126510	873490	000039	999961	14
47	135810	864190	135851	864149	000041	999959	13
48	144953	855047	144996	855004	000042	999958	12
49	153907	846093	153952	846048	000044	999956	11
50	8.162681	11.837319	8.162727	11.837273	10.000046	9.999954	10
51	171280	828720	171328	828672	000048	999952	9
52	179713	820287	179763	820237	000050	999950	8
53	187985	812015	188036	811964	000052	999948	7
54	196102	803898	196156	803844	000054	999946	6
55	8.204070	11.795930	8.204126	11.795874	10.000056	9.999944	5
56	211895	788105	211953	788047	000058	999942	4
57	219581	780419	219641	780359	000060	999940	3
58	227134	772866	227195	772805	000062	990938	2
59	234557	765443	234621	765379	000064	999936	1
60	241855	758145	241921	758079	000066	999934	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

89 Degrees.

1°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	8.241855	11.758145	8.241921	11.758079	10.000066	9.999934	60
1	249033	750967	249102	750898	000068	999932	59
2	256094	743906	256165	743835	000071	999929	58
3	263042	736958	263115	736885	000073	999927	57
4	269881	730119	269956	730044	000075	999925	56
5	8.276614	11.723386	8.276691	11.723309	10.000078	9.999922	55
6	283243	716757	283323	716677	000080	999920	54
7	289773	710227	289856	710144	000082	999918	53
8	296207	703793	296292	703708	000085	999915	52
9	302546	697454	302634	697366	000087	999913	51
10	8.308794	11.691206	8.308884	11.691116	10.000090	9.999910	50
11	314954	685046	315046	684954	000093	999907	49
12	321027	678973	321122	678878	000095	999905	48
13	327016	672984	327114	672886	000098	999902	47
14	332924	667076	333025	666975	000101	999899	46
15	8.338753	11.661247	8.338856	11.661144	10.000103	9.999897	45
16	344504	655496	344610	655390	000106	999894	44
17	350181	649819	350289	649711	000109	999891	43
18	355783	644217	355895	644105	000112	999888	42
19	361315	638685	361430	638570	000115	999885	41
20	8.366777	11.633223	8.366895	11.633105	10.000118	9.999882	40
21	372171	627829	372292	627708	000121	999879	39
22	377499	622501	377622	622378	000124	999876	38
23	382762	617238	382889	617111	000127	999873	37
24	387962	612038	388092	611908	000130	999870	36
25	8.393101	11.606899	8.393234	11.606766	10.000133	9.999867	35
26	398179	601821	398315	601685	000136	999864	34
27	403199	596801	403338	596662	000139	999861	33
28	408161	591839	408304	591696	000142	999858	32
29	413068	586932	413213	586787	000146	999854	31
30	8.417919	11.582081	8.418068	11.581932	10.000149	9.999851	30
31	422717	577283	422869	577131	000152	999848	29
32	427462	572538	427618	572382	000156	999844	28
33	432156	567844	432315	567685	000159	999841	27
34	436800	563200	436962	563038	000162	999838	26
35	8.441394	11.558606	8.441560	11.558440	10.000166	9.999834	25
36	445941	554059	446110	553890	000169	999831	24
37	450440	549560	450613	549387	000173	999827	23
38	454893	545107	455070	544930	000177	999823	22
39	459301	540699	459481	540519	000180	999820	21
40	8.463665	11.536335	8.463849	11.536151	10.000184	9.999816	20
41	467985	532015	468172	531828	000188	999812	19
42	472263	527737	472454	527546	000191	999809	18
43	476498	523502	476693	523307	000195	999805	17
44	480693	519307	480892	519108	000199	999801	16
45	8.484848	11.515152	8.485050	11.514950	10.000203	9.999797	15
46	488963	511037	489170	510830	000207	999793	14
47	493040	506960	493250	506750	000210	999790	13
48	497078	502922	497293	502707	000214	999786	12
49	501080	498920	501298	498702	000218	999782	11
50	8.505045	11.494955	8.505267	11.494733	10.000222	9.999778	10
51	508974	491026	509200	490800	000226	999774	9
52	512867	487133	513098	486902	000231	999769	8
53	516726	483274	516961	483039	000235	999765	7
54	520551	479449	520790	479210	000239	999761	6
55	8.524343	11.475657	8.524586	11.475414	10.000243	9.999757	5
56	528102	471898	528349	471651	000247	999753	4
57	531828	468172	532080	467920	000252	999748	3
58	535523	464477	535779	464221	000256	999744	2
59	539186	460814	539447	460553	000260	999740	1
60	542819	457181	543084	456916	000265	999735	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

88 Degrees.

2°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	8.542819	11.457181	8.543084	11.456916	10.000265	9.999735	60
1	546422	453578	546691	453309	000269	999731	59
2	549995	450005	550268	449732	000274	999726	58
3	553539	446461	553817	446183	000278	999722	57
4	557054	442946	557336	442664	000283	999717	56
5	8.560540	11.439460	8.560828	11.439172	10.000287	9.999713	55
6	563999	436001	564291	435709	000292	999708	54
7	567431	432569	567727	432273	000296	999704	53
8	570836	429164	571137	428863	000301	999699	52
9	574214	425786	574520	425480	000306	999694	51
10	8.577566	11.422434	8.577877	11.422123	10.000311	9.999689	50
11	580892	419108	581208	418792	000315	999685	49
12	584193	415807	584514	415486	000320	999680	48
13	587469	412531	587795	412205	000325	999675	47
14	590721	409279	591051	408949	000330	999670	46
15	8.593948	11.406052	8.594283	11.405717	10.000335	9.999665	45
16	597152	402848	597492	402508	000340	999660	44
17	600332	399668	600677	399323	000345	999655	43
18	603489	396511	603839	396161	000350	999650	42
19	606623	393377	606978	393022	000355	999645	11
20	8.609734	11.390266	8.610094	11.389906	10.000360	9.999640	40
21	612823	387177	613189	386811	000365	999635	39
22	615891	384109	616262	383738	000371	999629	38
23	618937	381063	619313	380687	000376	999624	37
24	621962	378038	622343	377657	000381	999619	36
25	8.624965	11.375035	8.625352	11.374648	10.000386	9.999614	35
26	627948	372052	628340	371660	000392	999608	34
27	630911	369089	631308	368692	000397	999603	33
28	633854	366146	634256	365744	000403	999597	32
29	636776	363224	637184	362816	000408	999592	31
30	8.639680	11.360320	8.640093	11.359907	10.000414	9.999586	30
31	642563	357437	642982	357018	000419	999581	29
32	645428	354572	645853	354147	000425	999575	28
33	648274	351726	648704	351296	000430	999570	27
34	651102	348898	651537	348463	000436	999564	26
35	8.653911	11.346089	8.654352	11.345648	10.000442	9.999558	25
36	656702	343298	657149	342851	000447	999553	24
37	659475	340525	659928	340072	000453	999547	23
38	662230	337770	662689	337311	000459	999541	22
39	664968	335032	665433	334567	000465	999535	21
40	8.667689	11.332311	8.668160	11.331840	10.000471	9.999529	20
41	670393	329607	670870	329130	000476	999524	19
42	673080	326920	673563	326437	000482	999518	18
43	675751	324249	676239	323761	000488	999512	17
44	678405	321595	678900	321100	000494	999506	16
45	8.681043	11.318957	8.681544	11.318456	10.000500	9.999500	15
46	683665	316335	684172	315828	000507	999493	14
47	686272	313728	686784	313216	000513	999487	13
48	688863	311137	689381	310619	000519	999481	12
49	691438	308562	691963	308037	000525	999475	11
50	8.693998	11.306002	8.694529	11.305471	10.000531	9.999469	10
51	696543	303457	697081	302919	000537	999463	9
52	699073	300927	699617	300383	000544	999456	8
53	701589	298411	702139	297861	000550	999450	7
54	704090	295910	704646	295354	000557	999443	6
55	8.706577	11.293423	8.707140	11.292860	10.000563	9.999437	5
56	709049	290951	709618	290382	000569	999431	4
57	711507	288493	712083	287917	000576	999424	3
58	713952	286048	714534	285466	000582	999418	2
59	716383	283617	716972	283028	000589	999411	1
60	718800	281200	719396	280604	000596	999404	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

87°

3°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	8.718800	11.281200	8.719396	11.280604	10.000596	9.999404	60
1	721204	278796	721806	278194	000602	999398	59
2	723595	276405	724204	275796	000609	999391	58
3	725972	274028	726588	273412	000616	999384	57
4	728337	271663	728959	271041	000622	999378	56
5	8.730688	11.269312	8.731317	11.268683	10.000629	9.999371	55
6	733027	266973	733663	266337	000636	999364	54
7	735354	264646	735996	264004	000643	999357	53
8	737667	262333	738317	261683	000650	999350	52
9	739969	260031	740626	259374	000657	999343	51
10	8.742259	11.257741	8.742922	11.257078	10.000664	9.999336	50
11	744536	255464	745207	254793	000671	999329	49
12	746802	253198	747479	252521	000678	999322	48
13	749055	250945	749740	250260	000685	999315	47
14	751297	248703	751989	248011	000692	999308	46
15	8.753528	11.246472	8.754227	11.245773	10.000699	9.999301	45
16	755747	244253	756453	243547	000706	999294	44
17	757955	242045	758668	241332	000714	999286	43
18	760151	239849	760872	239128	000721	999279	42
19	762337	237663	763065	236935	000728	999272	41
20	8.764511	11.235489	8.765246	11.234754	10.000735	9.999265	40
21	766675	233325	767417	232583	000743	999257	39
22	768828	231172	769578	230422	000750	999250	38
23	770970	229030	771727	228273	000758	999242	37
24	773101	226899	773866	226134	000765	999235	36
25	8.775223	11.224777	8.775995	11.224005	10.000773	9.999227	35
26	777333	222667	778114	221886	000780	999220	34
27	779434	220566	780222	219778	000788	999212	33
28	781524	218476	782320	217680	000795	999205	32
29	783605	216395	784408	215592	000803	999197	31
30	8.785675	11.214325	8.786486	11.213514	10.000811	9.999189	30
31	787736	212264	788554	211446	000819	999181	29
32	789787	210213	790613	209387	000826	999174	28
33	791828	208172	792662	207338	000834	999166	27
34	793859	206141	794701	205299	000842	999158	26
35	8.795881	11.204119	8 796731	11.203269	10.000850	9.999150	25
36	797894	202106	798752	201248	000858	999142	24
37	799897	200103	800763	199237	000866	999134	23
38	801892	198108	802765	197235	000874	999126	22
39	803876	196124	804758	195242	000882	999118	21
40	8.805852	11.194148	8.806742	11.193258	10.000890	9.999110	20
41	807819	192181	808717	191283	000898	999102	19
42	809777	190223	810683	189317	000906	999094	18
43	811726	188274	812641	187359	000914	999086	17
44	813667	186333	814589	185411	000923	999077	16
45	8.815599	11.184401	8.816529	11.183471	10.000931	9.999069	15
46	817522	182478	818461	181539	000939	999061	14
47	819436	180564	820384	179616	000947	999053	13
48	821343	178657	822298	177702	000956	999044	12
49	823240	176760	824205	175795	000964	999036	11
50	8.825130	11.174870	8.826103	11.173897	10.000973	9.999027	10
51	827011	172989	827992	172008	000981	999019	9
52	828884	171116	829874	170126	000990	999010	8
53	830749	169251	831748	168252	000998	999002	7
54	832607	167393	833613	166387	001007	998993	6
55	8.834456	11.165544	8.835471	11.164529	10.001016	9.998984	5
56	836297	163703	837321	162679	001024	998976	4
57	838130	161870	839163	160837	001033	998967	3
58	839956	160044	840998	159002	001042	998958	2
59	841774	158226	842825	157175	001050	998950	1
60	843585	156415	844644	155356	001059	998941	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

86°

TABLE III.—*Logarithmic Sines,*

4°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	8.843585	11.156415	8.844644	11.155356	10.001059	9.998941	60
1	845387	154613	846455	153545	001068	998932	59
2	847183	152817	848260	151740	001077	998923	58
3	848971	151029	850057	149943	001086	998914	57
4	850751	149249	851846	148154	001095	998905	56
5	8.852525	11.147475	8.853628	11.146372	10.001104	9.998896	55
6	854291	145709	855403	144597	001113	998887	54
7	856049	143951	857171	142829	001122	998878	53
8	857801	142199	858932	141068	001131	998869	52
9	859546	140454	860686	139314	001140	998860	51
10	8.861283	11.138717	8.862433	11.137567	10.001149	9.998851	50
11	863014	136986	864173	135827	001159	998841	49
12	864738	135262	865906	134094	001168	998832	48
13	866455	133545	867632	132368	001177	998823	47
14	868165	131835	869351	130649	001187	998813	46
15	8.869868	11.130132	8.871064	11.128936	10.001196	9.998804	45
16	871565	128435	872770	127230	001205	998795	44
17	873255	126745	874469	125531	001215	998785	43
18	874938	125062	876162	123838	001224	998776	42
19	876615	123385	877849	122151	001234	998766	41
20	8.878285	11.121715	8.879529	11.120471	10.001243	9.998757	40
21	879949	120051	881202	118798	001253	998747	39
22	881607	118393	882869	117131	001262	998738	38
23	883258	116742	884530	115470	001272	998728	37
24	884903	115097	886185	113815	001282	998718	36
25	8.886542	11.113458	8.887833	11.112167	10.001292	9.998708	35
26	888174	111826	889476	110524	001301	998699	34
27	889801	110199	891112	108888	001311	998689	33
28	891421	108579	892742	107258	001321	998679	32
29	893035	106965	894366	105634	001331	998669	31
30	8.894643	11.105357	8.895984	11.104016	10.001341	9.998659	30
31	896246	103754	897596	102404	001351	998649	29
32	897842	102158	899203	100797	001361	998639	28
33	899432	100568	900803	099197	001371	998629	27
34	901017	098983	902398	097602	001381	998619	26
35	8.902596	11.097404	8.903987	11.096013	10.001391	9.998609	25
36	904169	095831	905570	094430	001401	998599	24
37	905736	094264	907147	092853	001411	998589	23
38	907297	092703	908719	091281	001422	998578	22
39	908853	091147	910285	089715	001432	998568	21
40	8.910404	11.089596	8.911846	11.088154	10.001442	9.998558	20
41	911949	088051	913401	086599	001452	998548	19
42	913488	086512	914951	085049	001463	998537	18
43	915022	084978	916495	083505	001473	998527	17
44	916550	083450	918034	081966	001484	998516	16
45	8.918073	11.081927	8.919568	11.080432	10.001494	9.998506	15
46	919591	080409	921096	078904	001505	998495	14
47	921103	078897	922619	077381	001515	998485	13
48	922610	077390	924136	075864	001526	998474	12
49	924112	075888	925649	074351	001536	998464	11
50	8.925609	11.074391	8.927156	11.072844	10.001547	9.998453	10
51	927100	072900	928658	071342	001558	998442	9
52	928587	071413	930155	069845	001569	998431	8
53	930068	069932	931647	068353	001579	998421	7
54	931544	068456	933134	066866	001590	998410	6
55	8.933015	11.066985	8.934616	11.065384	10.001601	9.998399	5
56	934481	065519	936093	063907	001612	998388	4
57	935942	064058	937565	062435	001623	998377	3
58	937398	062602	939032	060968	001634	998366	2
59	938850	061150	940494	059506	001645	998355	1
60	940296	059704	941952	058048	001656	998344	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

85°

5°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	8.940296	11.059704	8.941952	11.058048	10.001656	9.998344	60
1	941738	058262	943404	056596	001667	998333	59
2	943174	056826	944852	055148	001678	998322	58
3	944606	055394	946295	053705	001689	998311	57
4	946034	053966	947734	052266	001700	998300	56
5	8.947456	11.052544	8.949168	11.050832	10.001711	9.998289	55
6	948874	051126	950597	049403	001723	998277	54
7	950287	049713	952021	047979	001734	998266	53
8	951696	048304	953441	046559	001745	998255	52
9	953100	046900	954856	045144	001757	998243	51
10	8.954499	11.045501	8.956267	11.043733	10.001768	9.998232	50
11	955894	044106	957674	042326	001780	998220	49
12	957284	042716	959075	040925	001791	998209	48
13	958670	041330	960473	039527	001803	998197	47
14	960052	039948	961866	038134	001814	998186	46
15	8.961429	11.038571	8.963255	11.036745	10.001826	9.998174	45
16	962801	037199	964639	035361	001837	998163	44
17	964170	035830	966019	033981	001849	998151	43
18	965534	034466	967394	032606	001861	998139	42
19	966893	033107	968766	031234	001872	998128	41
20	8.968249	11.031751	8.970133	11.029867	10.001884	9.998116	40
21	969600	030400	971496	028504	001896	998104	39
22	970947	029053	972855	027145	001908	998092	38
23	972289	027711	974209	025791	001920	998080	37
24	973628	026372	975560	024440	001932	998068	36
25	8.974962	11.025038	8.976906	11.023094	10.001944	9.998056	35
26	976293	023707	978248	021752	001956	998044	34
27	977619	022381	979586	020414	001968	998032	33
28	978941	021059	980921	019079	001980	998020	32
29	980259	019741	982251	017749	001992	998008	31
30	8.981573	11.018427	8.983577	11.016423	10.002004	9.997996	30
31	982883	017117	984899	015101	002016	997984	29
32	984189	015811	986217	013783	002028	997972	28
33	985491	014509	987532	012468	002041	997959	27
34	986789	013211	988842	011158	002053	997947	26
35	8.988083	11.011917	8.990149	11.009851	10.002065	9.997935	25
36	989374	010626	991451	008549	002078	997922	24
37	990660	009340	992750	007250	002090	997910	23
38	991943	008057	994045	005955	002103	997897	22
39	993222	006778	995337	004663	002115	997885	21
40	8.994497	11.005503	8.996624	11.003376	10.002128	9.997872	20
41	995768	004232	997908	002092	002140	997860	19
42	997036	002964	999188	000812	002153	997847	18
43	998299	001701	9.000465	10.999535	002165	997835	17
44	999560	000440	001738	998262	002178	997822	16
45	9.000816	10.999184	9.003007	10.996993	10.002191	9.997809	15
46	002069	997931	004272	995728	002203	997797	14
47	003318	996682	005534	994466	002216	997784	13
48	004563	995437	006792	993208	002229	997771	12
49	005805	994195	008047	991953	002242	997758	11
50	9.007044	10.992956	9.009298	10.990702	10.002255	9.997745	10
51	008278	991722	010546	989454	002268	997732	9
52	009510	990490	011790	988210	002281	997719	8
53	010737	989263	013031	986969	002294	997706	7
54	011962	988038	014268	985732	002307	997693	6
55	9.013182	10.986818	9.015502	10.984498	10.002320	9.997680	5
56	014400	985600	016732	983268	002333	997667	4
57	015613	984387	017959	982041	002346	997654	3
58	016824	983176	019183	980817	002359	997641	2
59	018031	981969	020403	979597	002372	997628	1
60	019235	980765	021620	978380	002386	997614	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

84°

6°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.019235	10.980765	9.021620	10.978380	10.002386	9.997614	60
1	020435	979565	022834	977166	002399	997601	59
2	021632	978368	024044	975956	002412	997588	58
3	022825	977175	025251	974749	002426	997574	57
4	024016	975984	026455	973545	002439	997561	56
5	9.025203	10.974797	9.027655	10.972345	10.002453	9.997547	55
6	026386	973614	028852	971148	002466	997534	54
7	027567	972433	030046	969954	002480	997520	53
8	028744	971256	031237	968763	002493	997507	52
9	029918	970082	032425	967575	002507	997493	51
10	9.031089	10.968911	9.033609	10.966391	10.002520	9.997480	50
11	032257	967743	034791	965209	002534	997466	49
12	033421	966579	035969	964031	002548	997452	48
13	034582	965418	037144	962856	002561	997439	47
14	035741	964259	038316	961684	002575	997425	46
15	9.036896	10.963104	9.039485	10.960515	10.002589	9.997411	45
16	038048	961952	040651	959349	002603	997397	44
17	039197	960803	041813	958187	002617	997383	43
18	040342	959658	042973	957027	002631	997369	42
19	041485	958515	044130	955870	002645	997355	41
20	9.042625	10.957375	9.045284	10.954716	10.002659	9.997341	40
21	043762	956238	046434	953566	002673	997327	39
22	044895	955105	047582	952418	002687	997313	38
23	046026	953974	048727	951273	002701	997299	37
24	047154	952846	049869	950131	002715	997285	36
25	9.048279	10.951721	9.051008	10.948992	10.002729	9.997271	35
26	049400	950600	052144	947856	002743	997257	34
27	050519	949481	053277	946723	002758	997242	33
28	051635	948365	054407	945593	002772	997228	32
29	052749	947251	055535	944465	002786	997214	31
30	9.053859	10.946141	9.056659	10.943341	10.002801	9.997199	30
31	054966	945034	057781	942219	002815	997185	29
32	056071	943929	058900	941100	002830	997170	28
33	057172	942828	060016	939984	002844	997156	27
34	058271	941729	061130	938870	002859	997141	26
35	9.059367	10.940633	9 062240	10.937760	10.002873	9.997127	25
36	060460	939540	063348	936652	002888	997112	24
37	061551	938449	064453	935547	002902	997098	23
38	062639	937361	065556	934444	002917	997083	22
39	063724	936276	066655	933345	002932	997068	21
40	9.064806	10.935194	9.067752	10.932248	10.002947	9.997053	20
41	065885	934115	068846	931154	002961	997039	19
42	066962	933038	069938	930062	002976	997024	18
43	068036	931964	071027	928973	002991	997009	17
44	069107	930893	072113	927887	003006	996994	16
45	9.070176	10.929824	9.073197	10.926803	10.003021	9.996979	15
46	071242	928758	074278	925722	003036	996964	14
47	072306	927694	075356	924644	003051	996949	13
48	073366	926634	076432	923568	003066	996934	12
49	074424	925576	077505	922495	003081	996919	11
50	9.075480	10.924520	9.078576	10.921424	10.003096	9.996904	10
51	076533	923467	079644	920356	003111	996889	9
52	077583	922417	080710	919290	003126	996874	8
53	078631	921369	081773	918227	003142	996858	7
54	079676	920324	082833	917167	003157	996843	6
55	9.080719	10.919281	9.083891	10.916109	10.003172	9.996828	5
56	081759	918241	084947	915053	003188	996812	4
57	082797	917203	086000	914000	003203	996797	3
58	083832	916168	087050	912950	003218	996782	2
59	084864	915136	088098	911902	003234	996766	1
60	085894	914106	089144	910856	003249	996751	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

83°

7°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.085894	10.914106	9.089144	10.910856	10.003249	9.996751	60
1	086922	913078	090187	909813	003265	996735	59
2	087947	912053	091228	908772	003280	996720	58
3	088970	911030	092266	907734	003296	996704	57
4	089990	910010	093302	906698	003312	996688	56
5	9.091008	10.908992	9.094336	10.905664	10.003327	9.996673	55
6	092024	907976	095367	904633	003343	996657	54
7	093037	906963	096395	903605	003359	996641	53
8	094047	905953	097422	902578	003375	996625	52
9	095056	904944	098446	901554	003390	996610	51
10	9.096062	10.903938	9.099468	10.900532	10.003406	9.996594	50
11	097065	902935	100487	899513	003422	996578	49
12	098066	901934	101504	898496	003438	996562	48
13	099065	900935	102519	897481	003454	996546	47
14	100062	899938	103532	896468	003470	996530	46
15	9.101056	10.898944	9.104542	10.895458	10.003486	9.996514	45
16	102048	897952	105550	894450	003502	996498	44
17	103037	896963	106556	893444	003518	996482	43
18	104025	895975	107559	892441	003535	996465	42
19	105010	894990	108560	891440	003551	996449	41
20	9.105992	10.894008	9.109559	10.890441	10.003567	9.996433	40
21	106973	893027	110556	889444	003583	996417	39
22	107951	892049	111551	888449	003600	996400	38
23	108927	891073	112543	887457	003616	996384	37
24	109901	890099	113533	886467	003632	996368	36
25	9.110873	10.889127	9.114521	10.885479	10.003649	9.996351	35
26	111842	888158	115507	884493	003665	996335	34
27	112809	887191	116491	883509	003682	996318	33
28	113774	886226	117472	882528	003698	996302	32
29	114737	885263	118452	881548	003715	996285	31
30	9.115698	10.884302	9.119429	10.880571	10.003731	9.996269	30
31	116656	883344	120404	879596	003748	996252	29
32	117613	882387	121377	878623	003765	996235	28
33	118567	881433	122348	877652	003781	996219	27
34	119519	880481	123317	876683	003798	996202	26
35	9.120469	10.879531	9.124284	10.875716	10.003815	9.996185	25
36	121417	878583	125249	874751	003832	996168	24
37	122362	877638	126211	873789	003849	996151	23
38	123306	876694	127172	872828	003866	996134	22
39	124248	875752	128130	871870	003883	996117	21
40	9.125187	10.874813	9.129087	10.870913	10.003900	9.996100	20
41	126125	873875	130041	869959	003917	996083	19
42	127060	872940	130994	869006	003934	996066	18
43	127993	872007	131944	868056	003951	996049	17
44	128925	871075	132893	867107	003968	996032	16
45	9.129854	10.870146	9.133839	10.866161	10.003985	9.996015	15
46	130781	869219	134784	865216	004002	995998	14
47	131706	868294	135726	864274	004020	995980	13
48	132630	867370	136667	863333	004037	995963	12
49	133551	866449	137605	862395	004054	995946	11
50	9.134470	10.865530	9.138542	10.861458	10.004072	9.995928	10
51	135387	864613	139476	860524	004089	995911	9
52	136303	863697	140409	859591	004106	995894	8
53	137216	862784	141340	858660	004124	995876	7
54	138128	861872	142269	857731	004141	995859	6
55	9.139037	10.860963	9.143196	10.856804	10.004159	9.995841	5
56	139944	860056	144121	855879	004177	995823	4
57	140850	859150	145044	854956	004194	995806	3
58	141754	858246	145966	854034	004212	995788	2
59	142655	857345	146885	853115	004229	995771	1
60	143555	856445	147803	852197	004247	995753	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

82°

8°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.143555	10.856445	9.147803	10.852197	10.004247	9.995753	60
1	144453	855547	148718	851282	004265	995735	59
2	145349	854651	149632	850368	004283	995717	58
3	146243	853757	150544	849456	004301	995699	57
4	147136	852864	151454	848546	004319	995681	56
5	9.148026	10.851974	9.152363	10.847637	10.004336	9.995664	55
6	148915	851085	153269	846731	004354	995646	54
7	149802	850198	154174	845826	004372	995628	53
8	150686	849314	155077	844923	004390	995610	52
9	151569	848431	155978	844022	004409	995591	51
10	9.152451	10.847549	9.156877	10.843123	10.004427	9.995573	50
11	153330	846670	157775	842225	004445	995555	49
12	154208	845792	158671	841329	004463	995537	48
13	155083	844917	159565	840435	004481	995519	47
14	155957	844043	160457	839543	004499	995501	46
15	9.156830	10.843170	9.161347	10.838653	10.004518	9.995482	45
16	157700	842300	162236	837764	004536	995464	44
17	158569	841431	163123	836877	004554	995446	43
18	159435	840565	164008	835992	004573	995427	42
19	160301	839699	164892	835108	004591	995409	41
20	9.161164	10.838836	9.165774	10.834226	10.004610	9.995390	40
21	162025	837975	166654	833346	004628	995372	39
22	162885	837115	167532	832468	004647	995353	38
23	163743	836257	168409	831591	004666	995334	37
24	164600	835400	169284	830716	004684	995316	36
25	9.165454	10.834546	9.170157	10.829843	10.004703	9.995297	35
26	166307	833693	171029	828971	004722	995278	34
27	167159	832841	171899	828101	004740	995260	33
28	168008	831992	172767	827233	004759	995241	32
29	168856	831144	173634	826366	004778	995222	31
30	9.169702	10.830298	9·174499	10.825501	10.004797	9.995203	30
31	170547	829453	175362	824638	004816	995184	29
32	171389	828611	176224	823776	004835	995165	28
33	172230	827770	177084	822916	004854	995146	27
34	173070	826930	177942	822058	004873	995127	26
35	9.173908	10.826092	9.178799	10.821201	10.004892	9.995108	25
36	174744	825256	179655	820345	004911	995089	24
37	175578	824422	180508	819492	004930	995070	23
38	176411	823589	181360	818640	004949	995051	22
39	177242	822758	182211	817789	004968	995032	21
40	9.178072	10.821928	9.183059	10.816941	10.004987	9.995013	20
41	178900	821100	183907	816093	005007	994993	19
42	179726	820274	184752	815248	005026	994974	18
43	180551	819449	185597	814403	005045	994955	17
44	181374	818626	186439	813561	005065	994935	16
45	9.182196	10.817804	9.187280	10.812720	10.005084	9.994916	15
46	183016	816984	188120	811880	005104	994896	14
47	183834	816166	188958	811042	005123	994877	13
48	184651	815349	189794	810206	005143	994857	12
49	185466	814534	190629	809371	005162	994838	11
50	9.186280	10.813720	9.191462	10.808538	10.005182	9.994818	10
51	187092	812908	192294	807706	005202	994798	9
52	187903	812097	193124	806876	005221	994779	8
53	188712	811288	193953	806047	005241	994759	7
54	189519	810481	194780	805220	005261	994739	6
55	9.190325	10.809675	9.195606	10.804394	10.005281	9.994719	5
56	191130	808870	196430	803570	005300	994700	4
57	191933	808067	197253	802747	005320	994680	3
58	192734	807266	198074	801926	005340	994660	2
59	193534	806466	198894	801106	005360	994640	1
60	194332	805668	199713	800287	005380	994620	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

81°

9°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.194332	10.805668	9.199713	10.800287	10.005380	9.994620	60
1	195129	804871	200529	799471	005400	994600	59
2	195925	804075	201345	798655	005420	994580	58
3	196719	803281	202159	797841	005440	994560	57
4	197511	802489	202971	797029	005460	994540	56
5	9.198302	10.801698	9.203782	10.796218	10.005481	9.994519	55
6	199091	800909	204592	795408	005501	994499	54
7	199879	800121	205400	794600	005521	994479	53
8	200666	799334	206207	793793	005541	994459	52
9	201451	798549	207013	792987	005562	994438	51
10	9.202234	10.797766	9.207817	10.792183	10.005582	9.994418	50
11	203017	796983	208619	791381	005602	994398	49
12	203797	796203	209420	790580	005623	994377	48
13	204577	795423	210220	789780	005643	994357	47
14	205354	794646	211018	788982	005664	994336	46
15	9.206131	10.793869	9.211815	10.788185	10.005684	9.994316	45
16	206906	793094	212611	787389	005705	994295	44
17	207679	792321	213405	786595	005726	994274	43
18	208452	791548	214198	785802	005746	994254	42
19	209222	790778	214989	785011	005767	994233	41
20	9.209992	10.790008	9.215780	10.784220	10.005788	9.994212	40
21	210760	789240	216568	783432	005809	994191	39
22	211526	788474	217356	782644	005829	994171	38
23	212291	787709	218142	781858	005850	994150	37
24	213055	786945	218926	781074	005871	994129	36
25	9.213818	10.786182	9.219710	10.780290	10.005892	9.994108	35
26	214579	785421	220492	779508	005913	994087	34
27	215338	784662	221272	778728	005934	994066	33
28	216097	783903	222052	777948	005955	994045	32
29	216854	783146	222830	777170	005976	994024	31
30	9.217609	10.782391	9.223607	10.776393	10.005997	9.994003	30
31	218363	781637	224382	775618	006018	993982	29
32	219116	780884	225156	774844	006040	993960	28
33	219868	780132	225929	774071	006061	993939	27
34	220618	779382	226700	773300	006082	993918	26
35	9.221367	10.778633	9.227471	10.772529	10.006104	9.993896	25
36	222115	777885	228239	771761	006125	993875	24
37	222861	777139	229007	770993	006146	993854	23
38	223606	776394	229773	770227	006168	993832	22
39	224349	775651	230539	769461	006189	993811	21
40	9.225092	10.774908	9.231302	10.768698	10.006211	9.993789	20
41	225833	774167	232065	767935	006232	993768	19
42	226573	773427	232826	767174	006254	993746	18
43	227311	772689	233586	766414	006275	993725	17
44	228048	771952	234345	765655	006297	993703	16
45	9.228784	10.771216	9.235103	10.764897	10.006319	9.993681	15
46	229518	770482	235859	764141	006340	993660	14
47	230252	769748	236614	763386	006362	993638	13
48	230984	769016	237368	762632	006384	993616	12
49	231714	768286	238120	761880	006406	993594	11
50	9.232444	10.767556	9.238872	10.761128	10.006428	9.993572	10
51	233172	766828	239622	760378	006450	993550	9
52	233899	766101	240371	759629	006472	993528	8
53	234625	765375	241118	758882	006494	993506	7
54	235349	764651	241865	758135	006516	993484	6
55	9.236073	10.763927	9.242610	10.757390	10.006538	9.993462	5
56	236795	763205	243354	756646	006560	993440	4
57	237515	762485	244097	755903	006582	993418	3
58	238235	761765	244839	755161	006604	993396	2
59	238953	761047	245579	754421	006626	993374	1
60	239670	760330	246319	753681	006649	993351	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

80°

10°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.239670	10.760330	9.246319	10.753681	10.006649	9.993351	60
1	240386	759614	247057	752943	006671	993329	59
2	241101	758899	247794	752206	006693	993307	58
3	241814	758186	248530	751470	006715	993285	57
4	242526	757474	249264	750736	006738	993262	56
5	9.243237	10.756763	9.249998	10.750002	10.006760	9.993240	55
6	243947	756053	250730	749270	006783	993217	54
7	244656	755344	251461	748539	006805	993195	53
8	245363	754637	252191	747809	006828	993172	52
9	246069	753931	252920	747080	006851	993149	51
10	9.246775	10.753225	9.253648	10.746352	10.006873	9.993127	50
11	247478	752522	254374	745626	006896	993104	49
12	248181	751819	255100	744900	006919	993081	48
13	248883	751117	255824	744176	006941	993059	47
14	249583	750417	256547	743453	006964	993036	46
15	9.250282	10.749718	9.257269	10.742731	10.006987	9.993013	45
16	250980	749020	257990	742010	007010	992990	44
17	251677	748323	258710	741290	007033	992967	43
18	252373	747627	259429	740571	007056	992944	42
19	253067	746933	260146	739854	007079	992921	41
20	9.253761	10.746239	9.260863	10.739137	10.007102	9.992898	40
21	254453	745547	261578	738422	007125	992875	39
22	255144	744856	262292	737708	007148	992852	38
23	255834	744166	263005	736995	007171	992829	37
24	256523	743477	263717	736283	007194	992806	36
25	9.257211	10.742789	9.264428	10.735572	10.007217	9.992783	35
26	257898	742102	265138	734862	007241	992759	34
27	258583	741417	265847	734153	007264	992736	33
28	259268	740732	266555	733445	007287	992713	32
29	259951	740049	267261	732739	007310	992690	31
30	9.260633	10.739367	9.267967	10.732033	10.007334	9.992666	30
31	261314	738686	268671	731329	007357	992643	29
32	261994	738006	269375	730625	007381	992619	28
33	262673	737327	270077	729923	007404	992596	27
34	263351	736649	270779	729221	007428	992572	26
35	9.264027	10.735973	9.271479	10.728521	10.007451	9.992549	25
36	264703	735297	272178	727822	007475	992525	24
37	265377	734623	272876	727124	007499	992501	23
38	266051	733949	273573	726427	007522	992478	22
39	266723	733277	274269	725731	007546	992454	21
40	9.267395	10.732605	9.274964	10.725036	10.007570	9.992430	20
41	268065	731935	275658	724342	007594	992406	19
42	268734	731266	276351	723649	007618	992382	18
43	269402	730598	277043	722957	007641	992359	17
44	270069	729931	277734	722266	007665	992335	16
45	9.270735	10.729265	9.278424	10.721576	10.007689	9.992311	15
46	271400	728600	279113	720887	007713	992287	14
47	272064	727936	279801	720199	007737	992263	13
48	272726	727274	280488	719512	007761	992239	12
49	273388	726612	281174	718826	007786	992214	11
50	9.274049	10.725951	9.281858	10.718142	10.007810	9.992190	10
51	274708	725292	282542	717458	007834	992166	9
52	275367	724633	283225	716775	007858	992142	8
53	276024	723976	283907	716093	007882	992118	7
54	276681	723319	284588	715412	007907	992093	6
55	9.277337	10.722663	9.285268	10.714732	10.007931	9.992069	5
56	277991	722009	285947	714053	007956	992044	4
57	278644	721356	286624	713376	007980	992030	3
58	279297	720703	287301	712699	008004	991996	2
59	279948	720052	287977	712023	008029	991971	1
60	280599	719401	288652	711348	008053	991947	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

79°

11°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.280599	10.719401	9.288652	10.711348	10.008053	9.991947	60
1	281248	718752	289326	710674	008078	991922	59
2	281897	718103	289999	710001	008103	991897	58
3	282544	717456	290671	709329	008127	991873	57
4	283190	716810	291342	708658	008152	991848	56
5	9.283836	10.716164	9.292013	10.707987	10.008177	9.991823	55
6	284480	715520	292682	707318	008201	991799	54
7	285124	714876	293350	706650	008226	991774	53
8	285766	714234	294017	705983	008251	991749	52
9	286408	713592	294684	705316	008276	991724	51
10	9.287048	10.712952	9.295349	10.704651	10.008301	9.991699	50
11	287687	712313	296013	703987	008326	991674	49
12	288326	711674	296677	703323	008351	991649	48
13	288964	711036	297339	702661	008376	991624	47
14	289600	710400	298001	701999	008401	991599	46
15	9.290236	10.709764	9.298662	10.701338	10.008426	9.991574	45
16	290870	709130	299322	700678	008451	991549	44
17	291504	708496	299980	700020	008476	991524	43
18	292137	707863	300638	699362	008502	991498	42
19	292768	707232	301295	698705	008527	991473	41
20	9.293399	10.706601	9.301951	10.698049	10.008552	9.991448	40
21	294029	705971	302607	697393	008578	991422	39
22	294658	705342	303261	696739	008603	991397	38
23	295286	704714	303914	696086	008628	991372	37
24	295913	704087	304567	695433	008654	991346	36
25	9.296539	10.703461	9.305218	10.694782	10.008679	9.991321	35
26	297164	702836	305869	694131	008705	991295	34
27	297788	702212	306519	693481	008730	991270	33
28	298412	701588	307168	692832	008756	991244	32
29	299034	700966	307815	692185	008782	991218	31
30	9.299655	10.700345	9.308463	10.691537	10.008807	9.991193	30
31	300276	699724	309109	690891	008833	991167	29
32	300895	699105	309754	690246	008859	991141	28
33	301514	698486	310398	689602	008885	991115	27
34	302132	697868	311042	688958	008910	991090	26
35	9.302748	10.697252	9.311685	10.688315	10.008936	9.991064	25
36	303364	696636	312327	687673	008962	991038	24
37	303979	696021	312967	687033	008988	991012	23
38	304593	695407	313608	686392	009014	990986	22
39	305207	694793	314247	685753	009040	990960	21
40	9.305819	10.694181	9.314885	10.685115	10.009066	9.990934	20
41	306430	693570	315523	684477	009092	990908	19
42	307041	692959	316159	683841	009118	990882	18
43	307650	692350	316795	683205	009145	990855	17
44	308259	691741	317430	682570	009171	990829	16
45	9.308867	10.691133	9.318064	10.681936	10.009197	9.990803	15
46	309474	690526	318697	681303	009223	990777	14
47	310080	689920	319329	680671	009250	990750	13
48	310685	689315	319961	680039	009276	990724	12
49	311289	688711	320592	679408	009303	990697	11
50	9.311893	10.688107	9.321222	10.678778	10.009329	9.990671	10
51	312495	687505	321851	678149	009356	990644	9
52	313097	686903	322479	677521	009382	990618	8
53	313698	686302	323106	676894	009409	990591	7
54	314297	685703	323733	676267	009435	990565	6
55	9.314897	10.685103	9.324358	10.675642	10.009462	9.990538	5
56	315495	684505	324983	675017	009489	990511	4
57	316092	683908	325607	674393	009515	990485	3
58	316689	683311	326231	673769	009542	990458	2
59	317284	682716	326853	673147	009569	990431	1
60	317879	682121	327475	672525	009596	990404	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

78°

TABLE III.—*Logarithmic Sines,*

12°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.317879	10.682121	9.327475	10.672525	10.009596	9.990404	60
1	318473	681527	328095	671905	009622	990378	59
2	319066	680934	328715	671285	009649	990351	58
3	319658	680342	329334	670666	009676	990324	57
4	320249	679751	329953	670047	009703	990297	56
5	9.320840	10.679160	9.330570	10.669430	10.009730	9.990270	55
6	321430	678570	331187	668813	009757	990243	54
7	322019	677981	331803	668197	009785	990215	53
8	322607	677393	332418	667582	009812	990188	52
9	323194	676806	333033	666967	009839	990161	51
10	9.323780	10.676220	9.333646	10.666354	10.009866	9.990134	50
11	324366	675634	334259	665741	009893	990107	49
12	324950	675050	334871	665129	009921	990079	48
13	325534	674466	335482	664518	009948	990052	47
14	326117	673883	336093	663907	009975	990025	46
15	9.326700	10.673300	9.336702	10.663298	10.010003	9.989997	45
16	327281	672719	337311	662689	010030	989970	44
17	327862	672138	337919	662081	010058	989942	43
18	328442	671558	338527	661473	010085	989915	42
19	329021	670979	339133	660867	010113	989887	41
20	9.329599	10.670401	9.339739	10.660261	10.010140	9.989860	40
21	330176	669824	340344	659656	010168	989832	39
22	330753	669247	340948	659052	010196	989804	38
23	331329	668671	341552	658448	010223	989777	37
24	331903	668097	342155	657845	010251	989749	36
25	9.332478	10.667522	9.342757	10.657243	10.010279	9.989721	35
26	333051	666949	343358	656642	010307	989693	34
27	333624	666376	343958	656042	010335	989665	33
28	334195	665805	344558	655442	010363	989637	32
29	334766	665234	345157	654843	010391	989609	31
30	9.335337	10.664663	9·345755	10.654245	10.010418	9.989582	30
31	335906	664094	346353	653647	010447	989553	29
32	336475	663525	346949	653051	010475	989525	28
33	337043	662957	347545	652455	010503	989497	27
34	337610	662390	348141	651859	010531	989469	26
35	9.338176	10.661824	9.348735	10.651265	10.010559	9.989441	25
36	338742	661258	349329	650671	010587	989413	24
37	339307	660693	349922	650078	010615	989385	23
38	339871	660129	350514	649486	010644	989356	22
39	340434	659566	351106	648894	010672	989328	21
40	9.340996	10.659004	9.351697	10.648303	10.010700	9.989300	20
41	341558	658442	352287	647713	010729	989271	19
42	342119	657881	352876	647124	010757	989243	18
43	342679	657321	353465	646535	010786	989214	17
44	343239	656761	354053	645947	010814	989186	16
45	9.343797	10.656203	9.354640	10.645360	10.010843	9.989157	15
46	344355	655645	355227	644773	010872	989128	14
47	344912	655088	355813	644187	010900	989100	13
48	345469	654531	356398	643602	010929	989071	12
49	346024	653976	356982	643018	010958	989042	11
50	9.346579	10.653421	9.357566	10.642434	10.010986	9.989014	10
51	347134	652866	358149	641851	011015	988985	9
52	347687	652313	358731	641269	011044	988956	8
53	348240	651760	359313	640687	011073	988927	7
54	348792	651208	359893	640107	011102	988898	6
55	9.349343	10.650657	9.360474	10.639526	10.011131	9.988869	5
56	349893	650107	361053	638947	011160	988840	4
57	350443	649557	361632	638368	011189	988811	3
58	350992	649008	362210	637790	011218	988782	2
59	351540	648460	362787	637213	011247	988753	1
60	352088	647912	363364	636636	011276	988724	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

77°

13°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.352088	10.647912	9.363364	10.636636	10.011276	9.988724	60
1	352635	647365	363940	636060	011305	988695	59
2	353181	646819	364515	635485	011334	988666	58
3	353726	646274	365090	634910	011364	988636	57
4	354271	645729	365664	634336	011393	988607	56
5	9.354815	10.645185	9.366237	10.633763	10.011422	9.988578	55
6	355358	644642	366810	633190	011452	988548	54
7	355901	644099	367382	632618	011481	988519	53
8	356443	643557	367953	632047	011511	988489	52
9	356984	643016	368524	631476	011540	988460	51
10	9.357524	10.642476	9.369094	10.630906	10.011570	9.988430	50
11	358064	641936	369663	630337	011599	988401	49
12	358603	641397	370232	629768	011629	988371	48
13	359141	640859	370799	629201	011658	988342	47
14	359678	640322	371367	628633	011688	988312	46
15	9.360215	10.639785	9.371933	10.628067	10.011718	9.988282	45
16	360751	639249	372499	627501	011748	988252	44
17	361287	638713	373064	626936	011777	988223	43
18	361822	638178	373629	626371	011807	988193	42
19	362356	637644	374193	625807	011837	988163	41
20	9.362889	10.637111	9.374756	10.625244	10.011867	9.988133	40
21	363422	636578	375319	624681	011897	988103	39
22	363954	636046	375881	624119	011927	988073	38
23	364485	635515	376442	623558	011957	988043	37
24	365016	634984	377003	622997	011987	988013	36
25	9.365546	10.634454	9.377563	10.622437	10.012017	9.987983	35
26	366075	633925	378122	621878	012047	987953	34
27	366604	633396	378681	621319	012078	987922	33
28	367131	632869	379239	620761	012108	987892	32
29	367659	632341	379797	620203	012138	987862	31
30	9.368185	10.631815	9.380354	10.619646	10.012168	9.987832	30
31	368711	631289	380910	619090	012199	987801	29
32	369236	630764	381466	618534	012229	987771	28
33	369761	630239	382020	617980	012260	987740	27
34	370285	629715	382575	617425	012290	987710	26
35	9.370808	10.629192	9.383129	10.616871	10.012321	9.987679	25
36	371330	628670	383682	616318	012351	987649	24
37	371852	628148	384234	615766	012382	987618	23
38	372373	627627	384786	615214	012412	987588	22
39	372894	627106	385337	614663	012443	987557	21
40	9.373414	10.626586	9.385888	10.614112	10.012474	9.987526	20
41	373933	626067	386438	613562	012504	987496	19
42	374452	625548	386987	613013	012535	987465	18
43	374970	625030	387536	612464	012566	987434	17
44	375487	624513	388084	611916	012597	987403	16
45	9.376003	10.623997	9.388631	10.611369	10.012628	9.987372	15
46	376519	623481	389178	610822	012659	987341	14
47	377035	622965	389724	610276	012690	987310	13
48	377549	622451	390270	609730	012721	987279	12
49	378063	621937	390815	609185	012752	987248	11
50	9.378577	10.621423	9.391360	10.608640	10.012783	9.987217	10
51	379089	620911	391903	608097	012814	987186	9
52	379601	620399	392447	607553	012845	987155	8
53	380113	619887	392989	607011	012876	987124	7
54	380624	619376	393531	606469	012908	987092	6
55	9.381134	10.618866	9.394073	10.605927	10.012939	9.987061	5
56	381643	618357	394614	605386	012970	987030	4
57	382152	617848	395154	604846	013002	986998	3
58	382661	617339	395694	604306	013033	986967	2
59	383168	616832	396233	603767	013064	986936	1
60	383675	616325	396771	603229	013096	986904	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

76°

14°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.383675	10.616325	9.396771	10.603229	10.013096	9.986904	60
1	384182	615818	397309	602691	013127	986873	59
2	384687	615313	397846	602154	013159	986841	58
3	385192	614808	398383	601617	013191	986809	57
4	385697	614303	398919	601081	013222	986778	56
5	9.386201	10.613799	9.399455	10.600545	10.013254	9.986746	55
6	386704	613296	399990	600010	013286	986714	54
7	387207	612793	400524	599476	013317	986683	53
8	387709	612291	401058	598942	013349	986651	52
9	388210	611790	401591	598409	013381	986619	51
10	9.388711	10.611289	9.402124	10.597876	10.013413	9.986587	50
11	389211	610789	402656	597344	013445	986555	49
12	389711	610289	403187	596813	013477	986523	48
13	390210	609790	403718	596282	013509	986491	47
14	390708	609292	404249	595751	013541	986459	46
15	9.391206	10.608794	9.404778	10.595222	10.013573	9.986427	45
16	391703	608297	405308	594692	013605	986395	44
17	392199	607801	405836	594164	013637	986363	43
18	392695	607305	406364	593636	013669	986331	42
19	393191	606809	406892	593108	013701	986299	41
20	9.393685	10.606315	9.407419	10.592581	10.013734	9.986266	40
21	394179	605821	407945	592055	013766	986234	39
22	394673	605327	408471	591529	013798	986202	38
23	395166	604834	408997	591003	013831	986169	37
24	395658	604342	409521	590479	013863	986137	36
25	9.396150	10.603850	9.410045	10.589955	10.013896	9.986104	35
26	396641	603359	410569	589431	013928	986072	34
27	397132	602868	411092	588908	013961	986039	33
28	397621	602379	411615	588385	013993	986007	32
29	398111	601889	412137	587863	014026	985974	31
30	9.398600	10.601400	9.412658	10.587342	10.014058	9.985942	30
31	399088	600912	413179	586821	014091	985909	29
32	399575	600425	413699	586301	014124	985876	28
33	400062	599938	414219	585781	014157	985843	27
34	400549	599451	414738	585262	014189	985811	26
35	9.401035	10.598965	9.415257	10.584743	10.014222	9.985778	25
36	401520	598480	415775	584225	014255	985745	24
37	402005	597995	416293	583707	014288	985712	23
38	402489	597511	416810	583190	014321	985679	22
39	402972	597028	417326	582674	014354	985646	21
40	9.403455	10.596545	9.417842	10.582158	10.014387	9.985613	20
41	403938	596062	418358	581642	014420	985580	19
42	404420	595580	418873	581127	014453	985547	18
43	404901	595099	419387	580613	014486	985514	17
44	405382	594618	419901	580099	014520	985480	16
45	9.405862	10.594138	9.420415	10.579585	10.014553	9.985447	15
46	406341	593659	420927	579073	014586	985414	14
47	406820	593180	421440	578560	014620	985380	13
48	407299	592701	421952	578048	014653	985347	12
49	407777	592223	422463	577537	014686	985314	11
50	9.408254	10.591746	9.422974	10.577026	10.014720	9.985280	10
51	408731	591269	423484	576516	014753	985247	9
52	409207	590793	423993	576007	014787	985213	8
53	409682	590318	424503	575497	014820	985180	7
54	410157	589843	425011	574989	014854	985146	6
55	9.410632	10.589368	9.425519	10.574481	10.014887	9.985113	5
56	411106	588894	426027	573973	014921	985079	4
57	411579	588421	426534	573466	014955	985045	3
58	412052	587948	427041	572959	014989	985011	2
59	412524	587476	427547	572453	015022	984978	1
60	412996	587004	428052	571948	015056	984944	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

75°

15°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.412996	10.587004	9.428052	10.571948	10.015056	9.984944	60
1	413467	586533	428557	571443	015090	984910	59
2	413938	586062	429062	570938	015124	984876	58
3	414408	585592	429566	570434	015158	984842	57
4	414878	585122	430070	569930	015192	984808	56
5	9.415347	10.584653	9.430573	10.569427	10.015226	9.984774	55
6	415815	584185	431075	568925	015260	984740	54
7	416283	583717	431577	568423	015294	984706	53
8	416751	583249	432079	567921	015328	984672	52
9	417217	582783	432580	567420	015362	984638	51
10	9.417684	10.582316	9.433080	10.566920	10.015397	9.984603	50
11	418150	581850	433580	566420	015431	984569	49
12	418615	581385	434080	565920	015465	984535	48
13	419079	580921	434579	565421	015500	984500	47
14	419544	580456	435078	564922	015534	984466	46
15	9.420007	10.579993	9.435576	10.564424	10.015568	9.984432	45
16	420470	579530	436073	563927	015603	984397	44
17	420933	579067	436570	563430	015637	984363	43
18	421395	578605	437067	562933	015672	984328	42
19	421857	578143	437563	562437	015706	984294	41
20	9.422318	10.577682	9.438059	10.561941	10.015741	9.984259	40
21	422778	577222	438554	561446	015776	984224	39
22	423238	576762	439048	560952	015810	984190	38
23	423697	576303	439543	560457	015845	984155	37
24	424156	575844	440036	559964	015880	984120	36
25	9.424615	10.575385	9.440529	10.559471	10.015915	9.984085	35
26	425073	574927	441022	558978	015950	984050	34
27	425530	574470	441514	558486	015985	984015	33
28	425987	574013	442006	557994	016019	983981	32
29	426443	573557	442497	557503	016054	983946	31
30	9.426899	10.573101	9.442988	10.557012	10.016089	9.983911	30
31	427354	572646	443479	556521	016125	983875	29
32	427809	572191	443968	556032	016160	983840	28
33	428263	571737	444458	555542	016195	983805	27
34	428717	571283	444947	555053	016230	983770	26
35	9.429170	10.570830	9.445435	10.554565	10.016265	9.983735	25
36	429623	570377	445923	554077	016300	983700	24
37	430075	569925	446411	553589	016336	983664	23
38	430527	569473	446898	553102	016371	983629	22
39	430978	569022	447384	552616	016406	983594	21
40	9.431429	10.568571	9.447870	10.552130	10.016442	9.983558	20
41	431879	568121	448356	551644	016477	983523	19
42	432329	567671	448841	551159	016513	983487	18
43	432778	567222	449326	550674	016548	983452	17
44	433226	566774	449810	550190	016584	983416	16
45	9.433675	10.566325	9.450294	10.549706	10.016619	9.983381	15
46	434122	565878	450777	549223	016655	983345	14
47	434569	565431	451260	548740	016691	983309	13
48	435016	564984	451743	548257	016727	983273	12
49	435462	564538	452225	547775	016762	983238	11
50	9.435908	10.564092	9.452706	10.547294	10.016798	9.983202	10
51	436353	563647	453187	546813	016834	983166	9
52	436798	563202	453668	546332	016870	983130	8
53	437242	562758	454148	545852	016906	983094	7
54	437686	562314	454628	545372	016942	983058	6
55	9.438129	10.561871	9.455107	10.544893	10.016978	9.983022	5
56	438572	561428	455586	544414	017014	982986	4
57	439014	560986	456064	543936	017050	982950	3
58	439456	560544	456542	543458	017086	982914	2
59	439897	560103	457019	542981	017122	982878	1
60	440338	559662	457496	542504	017158	982842	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

74°

16°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.440338	10.559662	9.457496	10.542504	10.017158	9.982842	60
1	440778	559222	457973	542027	017195	982805	59
2	441218	558782	458449	541551	017231	982769	58
3	441658	558342	458925	541075	017267	982733	57
4	442096	557904	459400	540600	017304	982696	56
5	9.442535	10.557465	9.459875	10.540125	10.017340	9.982660	55
6	442973	557027	460349	539651	017376	982624	54
7	443410	556590	460823	539177	017413	982587	53
8	443847	556153	461297	538703	017449	982551	52
9	444284	555716	461770	538230	017486	982514	51
10	9.444720	10.555280	9.462242	10.537758	10.017523	9.982477	50
11	445155	554845	462714	537286	017559	982441	49
12	445590	554410	463186	536814	017596	982404	48
13	446025	553975	463658	536342	017633	982367	47
14	446459	553541	464128	535872	017669	982331	46
15	9.446893	10.553107	9.464599	10.535401	10.017706	9.982294	45
16	447326	552674	465069	534931	017743	982257	44
17	447759	552241	465539	534461	017780	982220	43
18	448191	551809	466008	533992	017817	982183	42
19	448623	551377	466476	533524	017854	982146	41
20	9.449054	10.550946	9.466945	10.533055	10.017891	9.982109	40
21	449485	550515	467413	532587	017928	982072	39
22	449915	550085	467880	532120	017965	982035	38
23	450345	549655	468347	531653	018002	981998	37
24	450775	549225	468814	531186	018039	981961	36
25	9.451204	10.548796	9.469280	10.530720	10.018076	9.981924	35
26	451632	548368	469746	530254	018114	981886	34
27	452060	547940	470211	529789	018151	981849	33
28	452488	547512	470676	529324	018188	981812	32
29	452915	547085	471141	528859	018226	981774	31
30	9.453342	10.546658	9·471605	10.528395	10.018263	9.981737	30
31	453768	546232	472068	527932	018301	981699	29
32	454194	545806	472532	527468	018338	981662	28
33	454619	545381	472995	527005	018375	981625	27
34	455044	544956	473457	526543	018413	981587	26
35	9.455469	10.544531	9.473919	10.526081	10.018451	9.981549	25
36	455893	544107	474381	525619	018488	981512	24
37	456316	543684	474842	525158	018526	981474	23
38	456739	543261	475303	524697	018564	981436	22
39	457162	542838	475763	524237	018601	981399	21
40	9.457584	10.542416	9.476223	10.523777	10.018639	9.981361	20
41	458006	541994	476683	523317	018677	981323	19
42	458427	541573	477142	522858	018715	981285	18
43	458848	541152	477601	522399	018753	981247	17
44	459268	540732	478059	521941	018791	981209	16
45	9.459688	10.540312	9.478517	10.521483	10.018829	9.981171	15
46	460108	539892	478975	521025	018867	981133	14
47	460527	539473	479432	520568	018905	981095	13
48	460946	539054	479889	520111	018943	981057	12
49	461364	538636	480345	519655	018981	981019	11
50	9.461782	10.538218	9.480801	10.519199	10.019019	9.980981	10
51	462199	537801	481257	518743	019058	980942	9
52	462616	537384	481712	518288	019096	980904	8
53	463032	536968	482167	517833	019134	980866	7
54	463448	536552	482621	517379	019173	980827	6
55	9.463864	10.536136	9.483075	10.516925	10.019211	9.980789	5
56	464279	535721	483529	516471	019250	980750	4
57	464694	535306	483982	516018	019288	980712	3
58	465108	534892	484435	515565	019327	980673	2
59	465522	534478	484887	515113	019365	980635	1
60	465935	534065	485339	514661	019404	980596	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

73°

17°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.465935	10.534065	9.485339	10.514661	10.019404	9.980596	60
1	466348	533652	485791	514209	019442	980558	59
2	466761	533239	486242	513758	019481	980519	58
3	467173	532827	486693	513307	019520	980480	57
4	467585	532415	487143	512857	019558	980442	56
5	9.467996	10.532004	9.487593	10.512407	10.019597	9.980403	55
6	468407	531593	488043	511957	019636	980364	54
7	468817	531183	488492	511508	019675	980325	53
8	469227	530773	488941	511059	019714	980286	52
9	469637	530363	489390	510610	019753	980247	51
10	9.470046	10.529954	9.489838	10.510162	10.019792	9.980208	50
11	470455	529545	490286	509714	019831	980169	49
12	470863	529137	490733	509267	019870	980130	48
13	471271	528729	491180	508820	019909	980091	47
14	471679	528321	491627	508373	019948	980052	46
15	9.472086	10.527914	9.492073	10.507927	10.019988	9.980012	45
16	472492	527508	492519	507481	020027	979973	44
17	472898	527102	492965	507035	020066	979934	43
18	473304	526696	493410	506590	020105	979895	42
19	473710	526290	493854	506146	020145	979855	41
20	9.474115	10.525885	9.494299	10.505701	10.020184	9.979816	40
21	474519	525481	494743	505257	020224	979776	39
22	474923	525077	495186	504814	020263	979737	38
23	475327	524673	495630	504370	020303	979697	37
24	475730	524270	496073	503927	020342	979658	36
25	9.476133	10.523867	9.496515	10.503485	10.020382	9.979618	35
26	476536	523464	496957	503043	020421	979579	34
27	476938	523062	497399	502601	020461	979539	33
28	477340	522660	497841	502159	020501	979499	32
29	477741	522259	498282	501718	020541	979459	31
30	9.478142	10.521858	9.498722	10.501278	10.020580	9.979420	30
31	478542	521458	499163	500837	020620	979380	29
32	478942	521058	499603	500397	020660	979340	28
33	479342	520658	500042	499958	020700	979300	27
34	479741	520259	500481	499519	020740	979260	26
35	9.480140	10.519860	9.500920	10.499080	10.020780	9.979220	25
36	480539	519461	501359	498641	020820	979180	24
37	480937	519063	501797	498203	020860	979140	23
38	481334	518666	502235	497765	020900	979100	22
39	481731	518269	502672	497328	020941	979059	21
40	9.482128	10.517872	9.503109	10.496891	10.020981	9.979019	20
41	482525	517475	503546	496454	021021	978979	19
42	482921	517079	503982	496018	021061	978939	18
43	483316	516684	504418	495582	021102	978898	17
44	483712	516288	504854	495146	021142	978858	16
45	9.484107	10.515893	9.505289	10.494711	10.021183	9.978817	15
46	484501	515499	505724	494276	021223	978777	14
47	484895	515105	506159	493841	021263	978737	13
48	485289	514711	506593	493407	021304	978696	12
49	485682	514318	507027	492973	021345	978655	11
50	9.486075	10.513925	9.507460	10.492540	10.021385	9.978615	10
51	486467	513533	507893	492107	021426	978574	9
52	486860	513140	508326	491674	021467	978533	8
53	487251	512749	508759	491241	021507	978493	7
54	487643	512357	509191	490809	021548	978452	6
55	9.488034	10.511966	9.509622	10.490378	10.021589	9.978411	5
56	488424	511576	510054	489946	021630	978370	4
57	488814	511186	510485	489515	021671	978329	3
58	489204	510796	510916	489084	021712	978288	2
59	489593	510407	511346	488654	021753	978247	1
60	489982	510018	511776	488224	021794	978206	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

72°

18°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.489982	10.510018	9.511776	10.488224	10.021794	9.978206	60
1	490371	509629	512206	487794	021835	978165	59
2	490759	509241	512635	487365	021876	978124	58
3	491147	508853	513064	486936	021917	978083	57
4	491535	508465	513493	486507	021958	978042	56
5	9.491922	10.508078	9.513921	10.486079	10.021999	9.978001	55
6	492308	507692	514349	485651	022041	977959	54
7	492695	507305	514777	485223	022082	977918	53
8	493081	506919	515204	484796	022123	977877	52
9	493466	506534	515631	484369	022165	977835	51
10	9.493851	10.506149	9.516057	10.483943	10.022206	9.977794	50
11	494236	505764	516484	483516	022248	977752	49
12	494621	505379	516910	483090	022289	977711	48
13	495005	504995	517335	482665	022331	977669	47
14	495388	504612	517761	482239	022372	977628	46
15	9.495772	10.504228	9.518185	10.481815	10.022414	9.977586	45
16	496154	503846	518610	481390	022456	977544	44
17	496537	503463	519034	480966	022497	977503	43
18	496919	503081	519458	480542	022539	977461	42
19	497301	502699	519882	480118	022581	977419	41
20	9.497682	10.502318	9.520305	10.479695	10.022623	9.977377	40
21	498064	501936	520728	479272	022665	977335	39
22	498444	501556	521151	478849	022707	977293	38
23	498825	501175	521573	478427	022749	977251	37
24	499204	500796	521995	478005	022791	977209	36
25	9.499584	10.500416	9.522417	10.477583	10.022833	9.977167	35
26	499963	500037	522838	477162	022875	977125	34
27	500342	499658	523259	476741	022917	977083	33
28	500721	499279	523680	476320	022959	977041	32
29	501099	498901	524100	475900	023001	976999	31
30	9.501476	10.498524	9.524520	10.475480	10.023043	9.976957	30
31	501854	498146	524939	475061	023086	976914	29
32	502231	497769	525359	474641	023128	976872	28
33	502607	497393	525778	474222	023170	976830	27
34	502984	497016	526197	473803	023213	976787	26
35	9.503360	10.496640	9.526615	10.473385	10.023255	9.976745	25
36	503735	496265	527033	472967	023298	976702	24
37	504110	495890	527451	472549	023340	976660	23
38	504485	495515	527868	472132	023383	976617	22
39	504860	495140	528285	471715	023426	976574	21
40	9.505234	10.494766	9.528702	10.471298	10.023468	9.976532	20
41	505608	494392	529119	470881	023511	976489	19
42	505981	494019	529535	470465	023554	976446	18
43	506354	493646	529950	470050	023596	976404	17
44	506727	493273	530366	469634	023639	976361	16
45	9.507099	10.492901	9.530781	10.469219	10.023682	9.976318	15
46	507471	492529	531196	468804	023725	976275	14
47	507843	492157	531611	468389	023768	976232	13
48	508214	491786	532025	467975	023811	976189	12
49	508585	491415	532439	467561	023854	976146	11
50	9.508956	10.491044	9.532853	10.467147	10.023897	9.976103	10
51	509326	490674	533266	466734	023940	976060	9
52	509696	490304	533679	466321	023983	976017	8
53	510065	489935	534092	465908	024026	975974	7
54	510434	489566	534504	465496	024070	975930	6
55	9.510803	10.489197	9.534916	10.465084	10.024113	9.975887	5
56	511172	488828	535328	464672	024156	975844	4
57	511540	488460	535739	464261	024200	975800	3
58	511907	488093	536150	463850	024243	975757	2
59	512275	487725	536561	463439	024286	975714	1
60	512642	487358	536972	463028	024330	975670	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

71°

19°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.512642	10.487358	9.536972	10.463028	10.024330	9.975670	60
1	513009	486991	537382	462618	024373	975627	59
2	513375	486625	537792	462208	024417	975583	58
3	513741	486259	538202	461798	024461	975539	57
4	514107	485893	538611	461389	024504	975496	56
5	9.514472	10.485528	9.539020	10.460980	10.024548	9.975452	55
6	514837	485163	539429	460571	024592	975408	54
7	515202	484798	539837	460163	024635	975365	53
8	515566	484434	540245	459755	024679	975321	52
9	515930	484070	540653	459347	024723	975277	51
10	9.516294	10.483706	9.541061	10.458939	10.024767	9.975233	50
11	516657	483343	541468	458532	024811	975189	49
12	517020	482980	541875	458125	024855	975145	48
13	517382	482618	542281	457719	024899	975101	47
14	517745	482255	542688	457312	024943	975057	46
15	9.518107	10.481893	9.543094	10.456906	10.024987	9.975013	45
16	518468	481532	543499	456501	025031	974969	44
17	518829	481171	543905	456095	025075	974921	43
18	519190	480810	544310	455690	025120	974880	42
19	519551	480449	544715	455285	025164	974836	41
20	9.519911	10.480089	9.545119	10.454881	10.025208	9.974792	40
21	520271	479729	545524	454476	025252	974748	39
22	520631	479369	545928	454072	025297	974703	38
23	520990	479010	546331	453669	025341	974659	37
24	521349	478651	546735	453265	025386	974614	36
25	9.521707	10.478293	9.547138	10.452862	10.025430	9.974570	35
26	522066	477934	547540	452460	025475	974525	34
27	522424	477576	547943	452057	025519	974481	33
28	522781	477219	548345	451655	025564	974436	32
29	523138	476862	548747	451253	025609	974391	31
30	9.523495	10.476505	9.549149	10.450851	10.025653	9.974347	30
31	523852	476148	549550	450450	025698	974302	29
32	524208	475792	549951	450049	025743	974257	28
33	524564	475436	550352	449648	025788	974212	27
34	524920	475080	550752	449248	025833	974167	26
35	9.525275	10.474725	9.551152	10.448848	10.025878	9.974122	25
36	525630	474370	551552	448448	025923	974077	24
37	525984	474016	551952	448048	025968	974032	23
38	526339	473661	552351	447649	026013	973987	22
39	526693	473307	552750	447250	026058	973942	21
40	9.527046	10.472954	9.553149	10.446851	10.026103	9.973897	20
41	527400	472600	553548	446452	026148	973852	19
42	527753	472247	553946	446054	026193	973807	18
43	528105	471895	554344	445656	026239	973761	17
44	528458	471542	554741	445259	026284	973716	16
45	9.528810	10.471190	9.555139	10.444861	10.026329	9.973671	15
46	529161	470839	555536	444464	026375	973625	14
47	529513	470487	555933	444067	026420	973580	13
48	529864	470136	556329	443671	026465	973535	12
49	530215	469785	556725	443275	026511	973489	11
50	9.530565	10.469435	9.557121	10.442879	10.026556	9.973444	10
51	530915	469085	557517	442483	026602	973398	9
52	531265	478735	557913	442087	026648	973352	8
53	531614	468386	558308	441692	026693	973307	7
54	531963	468037	558702	441298	026739	973261	6
55	9.532312	10.467688	9.559097	10.440903	10.026785	9.973215	5
56	532661	467339	559491	440509	026831	973169	4
57	533009	466991	559885	440115	026876	973124	3
58	533357	466643	560279	439721	026922	973078	2
59	533704	466296	560673	439327	026968	973032	1
60	534052	465948	561066	438934	027014	972986	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

70°

20°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.534052	10.465948	9.561066	10.438934	10.027014	9.972986	60
1	534399	465601	561459	438541	027060	972940	59
2	534745	465255	561851	438149	027106	972894	58
3	535092	464908	562244	437756	027152	972848	57
4	535438	464562	562636	437364	027198	972802	56
5	9.535783	10.464217	9.563028	10.436972	10.027245	9.972755	55
6	536129	463871	563419	436581	027291	972709	54
7	536474	463526	563811	436189	027337	972663	53
8	536818	463182	564202	435798	027383	972617	52
9	537163	462837	564592	435408	027430	972570	51
10	9.537507	10.462493	9.564983	10.435017	10.027476	9.972524	50
11	537851	462149	565373	434627	027522	972478	49
12	538194	461806	565763	434237	027569	972431	48
13	538538	461462	566153	433847	027615	972385	47
14	538880	461120	566542	433458	027662	972338	46
15	9.539223	10.460777	9.566932	10.433068	10.027709	9.972291	45
16	539565	460435	567320	432680	027755	972245	44
17	539907	460093	567709	432291	027802	972198	43
18	540249	459751	568098	431902	027849	972151	42
19	540590	459410	568486	431514	027895	972105	41
20	9.540931	10.459069	9.568873	10.431127	10.027942	9.972058	40
21	541272	458728	569261	430739	027989	972011	39
22	541613	458387	569648	430352	028036	971964	38
23	541953	458047	570035	429965	028083	971917	37
24	542293	457707	570422	429578	028130	971870	36
25	9.542632	10.457368	9.570809	10.429191	10.028177	9.971823	35
26	542971	457029	571195	428805	028224	971776	34
27	543310	456690	571581	428419	028271	971729	33
28	543649	456351	571967	428033	028318	971682	32
29	543987	456013	572352	427648	028365	971635	31
30	9.544325	10.455675	9.572738	10.427262	10.028412	9.971588	30
31	544663	455337	573123	426877	028460	971540	29
32	545000	455000	573507	426493	028507	971493	28
33	545338	454662	573892	426108	028554	971446	27
34	545674	454326	574276	425724	028602	971398	26
35	9.546011	10.453989	9.574660	10.425340	10.028649	9.971351	25
36	546347	453653	575044	424956	028697	971303	24
37	546683	453317	575427	424573	028744	971256	23
38	547019	452981	575810	424190	028792	971208	22
39	547354	452646	576193	423807	028839	971161	21
40	9.547689	10.452311	9.576576	10.423424	10.028887	9.971113	20
41	548024	451976	576959	423041	028934	971066	19
42	548359	451641	577341	422659	028982	971018	18
43	548693	451307	577723	422277	029030	970970	17
44	549027	450973	578104	421896	029078	970922	16
45	9.549360	10.450640	9.578486	10.421514	10.029126	9.970874	15
46	549693	450307	578867	421133	029173	970827	14
47	550026	449974	579248	420752	029221	970779	13
48	550359	449641	579629	420371	029269	970731	12
49	550692	449308	580009	419991	029317	970683	11
50	9.551024	10.448976	9.580389	10.419611	10.029365	9.970635	10
51	551356	448644	580769	419231	029414	970586	9
52	551687	448313	581149	418851	029462	970538	8
53	552018	447982	581528	418472	029510	970490	7
54	552349	447651	581907	418093	029558	970442	6
55	9.552680	10.447320	9.582286	10.417714	10.029606	9.970394	5
56	553010	446990	582665	417335	029655	970345	4
57	553341	446659	583043	416957	029703	970297	3
58	553670	446330	583422	416578	029751	979249	2
59	554000	446000	583800	416200	029800	970200	1
60	554329	445671	584177	415823	029848	970152	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

69°

21°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.554329	10.445671	9.584177	10.415823	10.029848	9.970152	60
1	554658	445342	584555	415445	029897	970103	59
2	554987	445013	584932	415068	029945	970055	58
3	555315	444685	585309	414691	029994	970006	57
4	555643	444357	585686	414314	030043	969957	56
5	9.555971	10.444029	9.586062	10.413938	10.030091	9.969909	55
6	556299	443701	586439	413561	030140	969860	54
7	556626	443374	586815	413185	030189	969811	53
8	556953	443047	587190	412810	030238	969762	52
9	557280	442720	587566	412434	030286	969714	51
10	9.557606	10.442394	9.587941	10.412059	10.030335	9.969665	50
11	557932	442068	588316	411684	030384	969616	49
12	558258	441742	588691	411309	030433	969567	48
13	558583	441417	589066	410934	030482	969518	47
14	558909	441091	589440	410560	030531	969469	46
15	9.559234	10.440766	9.589814	10.410186	10.030580	9.969420	45
16	559558	440442	590188	409812	030630	969370	44
17	559883	440117	590562	409438	030679	969321	43
18	560207	439792	590935	409065	030728	969272	42
19	560531	439469	591308	408692	030777	969223	41
20	9.560855	10.439145	9.591681	10.408319	10.030827	9.969173	40
21	561178	438822	592054	407946	030876	969124	39
22	561501	438499	592426	407574	030925	969075	38
23	561824	438176	592798	407202	030975	969025	37
24	562146	437854	593171	406829	031024	968976	36
25	9.562468	10.437532	9.593542	10.406458	10.031074	9.968926	35
26	562790	437210	593914	406086	031123	968877	34
27	563112	436888	594285	405715	031173	968827	33
28	563433	436567	594656	405344	031223	968777	32
29	563755	436245	595027	404973	031272	968728	31
30	9.564075	10.435925	9·595398	10.404602	10.031322	9.968678	30
31	564396	435604	595768	404232	031372	968628	29
32	564716	435284	596138	403862	031422	968578	28
33	565036	434964	596508	403492	031472	968528	27
34	565356	434644	596878	403122	031521	968479	26
35	9.565676	10.434324	9.597247	10.402753	10.031571	9.968429	25
36	565995	434005	597616	402384	031621	968379	24
37	566314	433686	597985	402015	031671	968329	23
38	566632	433368	598354	401646	031722	968278	22
39	566951	433049	598722	401278	031772	968228	21
40	9.567269	10.432731	9.599091	10.400909	10.031822	9.968178	20
41	567587	432413	599459	400541	031872	968128	19
42	567904	432096	599827	400173	031922	968078	18
43	568222	431778	600194	399806	031973	968027	17
44	568539	431461	600562	399438	032023	967977	16
45	9.568856	10.431144	9.600929	10.399071	10.032073	9.967927	15
46	569172	430828	601296	398704	032124	967876	14
47	569488	430512	601662	398338	032174	967826	13
48	569804	430196	602029	397971	032225	967775	12
49	570120	429880	602395	397605	032275	967725	11
50	9.570435	10.429565	9.602761	10.397239	10.032326	9.967674	10
51	570751	429249	603127	396873	032376	967624	9
52	571066	428934	603493	396507	032427	967573	8
53	571380	428620	603858	396142	032478	967522	7
54	571695	428305	604223	395777	032529	967471	6
55	9.572009	10.427991	9.604588	10.395412	10.032579	9.967421	5
56	572323	427677	604953	395047	032630	967370	4
57	572636	427364	605317	394683	032681	967319	3
58	572950	427050	605682	394318	032732	967268	2
59	573263	426737	606046	393954	032783	967217	1
60	573575	426425	606410	393590	032834	967166	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

68°

22°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.573575	10.426425	9.606410	10.393590	10.032834	9.967166	60
1	573888	426112	606773	393227	032885	967115	59
2	574200	425800	607137	392863	032936	967064	58
3	574512	425488	607500	392500	032987	967013	57
4	574824	425176	607863	392137	033039	966961	56
5	9.575136	10.424864	9.608225	10.391775	10.033090	9.966910	55
6	575447	424553	608588	391412	033141	966859	54
7	575758	424242	608950	391050	033192	966808	53
8	576069	423931	609312	390688	033244	966756	52
9	576379	423621	609674	390326	033295	966705	51
10	9.576689	10.423311	9.610036	10.389964	10.033347	9.966653	50
11	576999	423001	610397	389603	033398	966602	49
12	577309	422691	610759	389241	033450	966550	48
13	577618	422382	611120	388880	033501	966499	47
14	577927	422073	611480	388520	033553	966447	46
15	9.578236	10.421764	9.611841	10.388159	10.033605	9.966395	45
16	578545	421455	612201	387799	033656	966344	44
17	578853	421147	612561	387439	033708	966292	43
18	579162	420838	612921	387079	033760	966240	42
19	579470	420530	613281	386719	033812	966188	41
20	9.579777	10.420223	9.613641	10.386359	10.033864	9.966136	40
21	580085	419915	614000	386000	033915	966085	39
22	580392	419608	614359	385641	033967	966033	38
23	580699	419301	614718	385282	034019	965981	37
24	581005	418995	615077	384923	034071	965929	36
25	9.581312	10.418688	9.615435	10.384565	10.034124	9.965876	35
26	581618	418382	615793	384207	034176	965824	34
27	581924	418076	616151	383849	034228	965772	33
28	582229	417771	616509	383491	034280	965720	32
29	582535	417465	616867	383133	034332	965668	31
30	9.582840	10.417160	9.617224	10.382776	10.034385	9.965615	30
31	583145	416855	617582	382418	034437	965563	29
32	583449	416551	617939	382061	034489	965511	28
33	583754	416246	618295	381705	034542	965458	27
34	584058	415942	618652	381348	034594	965406	26
35	9.584361	10.415639	9.619008	10.380992	10.034647	9.965353	25
36	584665	415335	619364	380636	034699	965301	24
37	584968	415032	619721	380279	034752	965248	23
38	585272	414728	620076	379924	034805	965195	22
39	585574	414426	620432	379568	034857	965143	21
40	9.585877	10.414123	9.620787	10.379213	10.034910	9.965090	20
41	586179	413821	621142	378858	034963	965037	19
42	586482	413518	621497	378503	035016	964984	18
43	586783	413217	621852	378148	035069	964931	17
44	587085	412915	622207	377793	035121	964879	16
45	9.587386	10.412614	9.622561	10.377439	10.035174	9.964826	15
46	587688	412312	622915	377085	035227	964773	14
47	587989	412011	623269	376731	035280	964720	13
48	588289	411711	623623	376377	035334	964666	12
49	588590	411410	623976	376024	035387	964613	11
50	9.588890	10.411110	9.624330	10.375670	10.035440	9.964560	10
51	589190	410810	624683	375317	035493	964507	9
52	589489	410511	625036	374964	035546	964454	8
53	589789	410211	625388	374612	035600	964400	7
54	590088	409912	625741	374259	035653	964347	6
55	9.590387	10.409613	9.626093	10.373907	10.035706	9.964294	5
56	590686	409314	626445	373555	035760	964240	4
57	590984	409016	626797	373203	035813	964187	3
58	591282	408718	627149	372851	035867	964133	2
59	591580	408420	627501	372499	035920	964080	1
60	591878	408122	627852	372148	035974	964026	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

67°

23°

′	Sine.	Cosec.	Tang.	Cotang.	Secant	Cosine.	′
0	9.591878	10.408122	9.627852	10.372148	10.035974	9.964026	60
1	592176	407824	628203	371797	036028	963972	59
2	592473	407527	628554	371446	036081	963919	58
3	592770	407230	628905	371095	036135	963865	57
4	593067	406933	629255	370745	036189	963811	56
5	9.593363	10.406637	9.629606	10.370394	10.036243	9.963757	55
6	593659	406341	629956	370044	036296	963704	54
7	593955	406045	630306	369694	036350	963650	53
8	594251	405749	630656	369344	036404	963596	52
9	594547	405453	631005	368995	036458	963542	51
10	9.594842	10.405158	9.631355	10.368645	10.036512	9.963488	50
11	595137	404863	631704	368296	036566	963434	49
12	595432	404568	632053	367947	036621	963379	48
13	595727	404273	632401	367599	036675	963325	47
14	596021	403979	632750	367250	036729	963271	46
15	9.596315	10.403685	9.633098	10.366902	10.036783	9.963217	45
16	596609	403391	633447	366553	036837	963163	44
17	596903	403097	633795	366205	036892	963108	43
18	597196	402804	634143	365857	036946	963054	42
19	597490	402510	634490	365510	037001	962999	41
20	9.597783	10.402217	9.634838	10.365162	10.037055	9.962945	40
21	598075	401925	635185	364815	037110	962890	39
22	598368	401632	635532	364468	037164	962836	38
23	598660	401340	635879	364121	037219	962781	37
24	598952	401048	636226	363774	037273	962727	36
25	9.599244	10.400756	9.636572	10.363428	10.037328	9.962672	35
26	599536	400464	636919	363081	037383	962617	34
27	599827	400173	637265	362735	037438	962562	33
28	600118	399882	637611	362389	037492	962508	32
29	600409	399591	637956	362044	037547	962453	31
30	9.600700	10.399300	9.638302	10.361698	10.037602	9.962398	30
31	600990	399010	638647	361353	037657	962343	29
32	601280	398720	638992	361008	037712	962288	28
33	601570	398430	639337	360663	037767	962233	27
34	601860	398140	639682	360318	037822	962178	26
35	9.602150	10.397850	9.640027	10.359973	10.037877	9.962123	25
36	602439	397561	640371	359629	037933	962067	24
37	602728	397272	640716	359284	037988	962012	23
38	603017	396983	641060	358940	038043	961957	22
39	603305	396695	641404	358596	038098	961902	21
40	9.603594	10.396406	9.641747	10.358253	10.038154	9.961846	20
41	603882	396118	642091	357909	038209	961791	19
42	604170	395830	642434	357566	038265	961735	18
43	604457	395543	642777	357223	038320	961680	17
44	604745	395255	643120	356880	038376	961624	16
45	9.605032	10.394968	9.643463	10.356537	10.038431	9.961569	15
46	605319	394681	643806	356194	038487	961513	14
47	605606	394394	644148	355852	038542	961458	13
48	605892	394108	644490	355510	038598	961402	12
49	606179	393821	644832	355168	038654	961346	11
50	9.606465	10.393535	9.645174	10.354826	10.038710	9.961290	10
51	606751	393249	645516	354484	038765	961235	9
52	607036	392964	645857	354143	038821	961179	8
53	607322	392678	646199	353801	038877	961123	7
54	607607	392393	646540	353460	038933	961067	6
55	9.607892	10.392108	9.646881	10.353119	10.038989	9.961011	5
56	608177	391823	647222	352778	039045	960955	4
57	608461	391539	647562	352438	039101	960899	3
58	608745	391255	647903	352097	039157	960843	2
59	609029	390971	648243	351757	039214	960786	1
60	609313	390687	648583	351417	039270	960730	0
′	Cosine.	Secant	Cotang.	Tang.	Cosec.	Sine.	′

66°

TABLE III.—*Logarithmic Sines,*

24°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.609313	10.390687	9.648583	10.351417	10.039270	9.960730	60
1	609597	390403	648923	351077	039326	960674	59
2	609880	390120	649263	350737	039382	960618	58
3	610164	389836	649602	350398	039439	960561	57
4	610447	389553	649942	350058	039495	960505	56
5	9.610729	10.389271	9.650281	10.349719	10.039552	9.960448	55
6	611012	388988	650620	349380	039608	960392	54
7	611294	388706	650959	349041	039665	960335	53
8	611576	388424	651297	348703	039721	960279	52
9	611858	388142	651636	348364	039778	960222	51
10	9.612140	10.387860	9.651974	10.348026	10.039835	9.960165	50
11	612421	387579	652312	347688	039891	960109	49
12	612702	387298	652650	347350	039948	960052	48
13	612983	387017	652988	347012	040005	959995	47
14	613264	386736	653326	346674	040062	959938	46
15	9.613545	10.386455	9.653663	10.346337	10.040118	9.959882	45
16	613825	386175	654000	346000	040175	959825	44
17	614105	385895	654337	345663	040232	959768	43
18	614385	385615	654674	345326	040289	959711	42
19	614665	385335	655011	344989	040346	959654	41
20	9.614944	10.385056	9.655348	10.344652	10.040404	9.959596	40
21	615223	384777	655684	344316	040461	959539	39
22	615502	384498	656020	343980	040518	959482	38
23	615781	384219	656356	343644	040575	959425	37
24	616060	383940	656692	343308	040632	959368	36
25	9.616338	10.383662	9.657028	10.342972	10.040690	9.959310	35
26	616616	383384	657364	342636	040747	959253	34
27	616894	383106	657699	342301	040805	959195	33
28	617172	382828	658034	341966	040862	959138	32
29	617450	382550	658369	341631	040920	959080	31
30	9.617727	10.382273	9·658704	10.341296	10.040977	9.959023	30
31	618004	381996	659039	340961	041035	958965	29
32	618281	381719	659373	340627	041092	958908	28
33	618558	381442	659708	340292	041150	958850	27
34	618834	381166	660042	339958	041208	958792	26
35	9.619110	10.380890	9.660376	10.339624	10.041266	9.958734	25
36	619386	380614	660710	339290	041323	958677	24
37	619662	380338	661043	338957	041381	958619	23
38	619938	380062	661377	338623	041439	958561	22
39	620213	379787	661710	338290	041497	958503	21
40	9.620488	10.379512	9.662043	10.337957	10.041555	9.958445	20
41	620763	379237	662376	337624	041613	958387	19
42	621038	378962	662709	337291	041671	958329	18
43	621313	378687	663042	336958	041729	958271	17
44	621587	378413	663375	336625	041787	958213	16
45	9.621861	10.378139	9.663707	10.336293	10.041846	9.958154	15
46	622135	377865	664039	335961	041904	958096	14
47	622409	377591	664371	335629	041962	958038	13
48	622682	377318	664703	335297	042021	957979	12
49	622956	377044	665035	334965	042079	957921	11
50	9.623229	10.376771	9.665366	10.334634	10.042137	9.957863	10
51	623502	376498	665697	334303	042196	957804	9
52	623774	376226	666029	333971	042254	957746	8
53	624047	375953	666360	333640	042313	957687	7
54	624319	375681	666691	333309	042372	957628	6
55	9.624591	10.375409	9.667021	10.332979	10.042430	9.957570	5
56	624863	375137	667352	332648	042489	957511	4
57	625135	374865	667682	332318	042548	957452	3
58	625406	374594	668013	331987	042607	957393	2
59	625677	374323	668343	331657	042665	957335	1
60	625948	374052	668672	331328	042724	957276	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

65°

25°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.625948	10.374052	9.668673	10.331327	10.042724	9.957276	60
1	626219	373781	669002	330998	042783	957217	59
2	626490	373510	669332	330668	042842	957158	58
3	626760	373240	669661	330339	042901	957099	57
4	627030	372970	669991	330009	042960	957040	56
5	9.627300	10.372700	9.670320	10.329680	10.043019	9.956981	55
6	627570	372430	670649	329351	043079	956921	54
7	627840	372160	670977	329023	043138	956862	53
8	628109	371891	671306	328694	043197	956803	52
9	628378	371622	671634	328366	043256	956744	51
10	9.628647	10.371353	9.671963	10.328037	10.043316	9.956684	50
11	628916	371084	672291	327709	043375	956625	49
12	629185	370815	672619	327381	043434	956566	48
13	629453	370574	672947	327053	043494	956506	47
14	629721	370279	673274	326726	043553	956447	46
15	9.629989	10.370011	9.673602	10.326398	10.043613	9.956387	45
16	630257	369743	673929	326071	043673	956327	44
17	630524	369476	674257	325743	043732	956268	43
18	630792	369208	674584	325416	043792	956208	42
19	631059	368941	674910	325090	043852	956148	41
20	9.631326	10.368674	9.675237	10.324763	10.043911	9.956089	40
21	631593	368407	675564	324436	043971	956029	39
22	631859	368141	675890	324110	044031	955969	38
23	632125	367875	676217	323783	044091	955909	37
24	632392	367608	676543	323457	044151	955849	36
25	9.632658	10.367342	9.676869	10.323131	10.044211	9.955789	35
26	632923	367077	677194	322806	044271	955729	34
27	633189	366811	677520	322480	044331	955669	33
28	633454	366546	677846	322154	044391	955609	32
29	633719	366281	678171	321829	044452	955548	31
30	9.633984	10.366016	9.678496	10.321504	10.044512	9.955488	30
31	634249	365751	678821	321179	044572	955428	29
32	634514	365486	679146	320854	044632	955368	28
33	634778	365222	679471	320529	044693	955307	27
34	635042	364958	679795	320205	044753	955247	26
35	9.635306	10.364694	9.680120	10.319880	10.044814	9.955186	25
36	635570	364430	680444	319556	044874	955126	24
37	635834	364166	680768	319232	044935	955065	23
38	636097	363903	681092	318908	044995	955005	22
39	636360	363640	681416	318584	045056	954944	21
40	9.636623	10.363377	9.681740	10.318260	10.045117	9.954883	20
41	636886	363114	682063	317937	045177	954823	19
42	637148	362852	682387	317613	045238	954762	18
43	637411	362589	682710	317290	045299	954701	17
44	637673	362327	683033	316967	045360	954640	16
45	9.637935	10.362065	9.683356	10.316644	10.045421	9.954579	15
46	638197	361803	683679	316321	045482	954518	14
47	638458	361542	684001	315999	045543	954457	13
48	638720	361280	684324	315676	045604	954396	12
49	638981	361019	684646	315354	045665	954335	11
50	9.639242	10.360758	9.684968	10.315032	10.045726	9.954274	10
51	639503	360497	685290	314710	045787	954213	9
52	639764	360236	685612	314388	045848	954152	8
53	640024	359976	685934	314066	045910	954090	7
54	640284	359716	686255	313745	045971	954029	6
55	9.640544	10.359456	9.686577	10.313423	10.046032	9.953968	5
56	640804	359196	686898	313102	046094	953906	4
57	641064	358936	687219	312781	046155	953845	3
58	641324	358676	687540	312460	046217	953783	2
59	641583	358417	687861	312139	046278	953722	1
60	641842	358158	688182	311818	046340	953660	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

64°

TABLE III.—*Logarithmic Sines,*

26°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.641842	10.358158	9.688182	10.311818	10.046340	9.953660	60
1	642101	357899	688502	311498	046401	953599	59
2	642360	357640	688823	311177	046463	953537	58
3	642618	357382	689143	310857	046525	953475	57
4	642877	357123	689463	310537	046587	953413	56
5	9.643135	10.356865	9.689783	10.310217	10.046648	9.953352	55
6	643393	356607	690103	309897	046710	953290	54
7	643650	356350	690423	309577	046772	953228	53
8	643908	356092	690742	309258	046834	953166	52
9	644165	355835	691062	306938	046896	953104	51
10	9.644423	10.355577	9.691381	10.308619	10.046958	9.953042	50
11	644680	355320	691700	308300	047020	952980	49
12	644936	355064	692019	307981	047082	952918	48
13	645193	354807	692338	307662	047145	952855	47
14	645450	354550	692656	307344	047207	952793	46
15	9.645706	10.354294	9.692975	10.307025	10.047269	9.952731	45
16	645962	354038	693293	306707	047331	952669	44
17	646218	353782	693612	306388	047394	952606	43
18	646474	353526	693930	306070	047456	952544	42
19	646729	353271	694248	305752	047519	952481	41
20	9.646984	10.353016	9.694566	10.305434	10.047581	9.952419	40
21	647240	352760	694883	305117	047644	952356	39
22	647494	352506	695201	304799	047706	952294	38
23	647749	352251	695518	304482	047769	952231	37
24	648004	351996	695836	304164	047832	952168	36
25	9.648258	10.351742	9.696153	10.303847	10.047894	9.952106	35
26	648512	351488	696470	303530	047957	952043	34
27	648766	351234	696787	303213	048020	951980	33
28	649020	350980	697103	302897	048083	951917	32
29	649274	350726	697420	302580	048146	951854	31
30	9.649527	10.350473	9.697736	10.302264	10.048209	9.951791	30
31	649781	350219	698053	301947	048272	951728	29
32	650034	349966	698369	301631	048335	951665	28
33	650287	349713	698685	301315	048398	951602	27
34	650539	349461	699001	300999	048461	951539	26
35	9.650792	10.349208	9.699316	10.300684	10.048524	9.951476	25
36	651044	348956	699632	300368	048588	951412	24
37	651297	348703	699947	300053	048651	951349	23
38	651549	348451	700263	299737	048714	951286	22
39	651800	348200	700578	299422	048778	951222	21
40	9.652052	10.347948	9.700893	10.299107	10.048841	9.951159	20
41	652304	347696	701208	298792	048904	951096	19
42	652555	347445	701523	298477	048968	951032	18
43	652806	347194	701837	298163	049032	950968	17
44	653057	346943	702152	297848	049095	950905	16
45	9.653308	10.346692	9.702466	10.297534	10.049159	9.950841	15
46	653558	346442	702780	297220	049222	950778	14
47	653808	346192	703095	296905	049286	950714	13
48	654059	345941	703409	296591	049350	950650	12
49	654309	345691	703723	296277	049414	950586	11
50	9.654558	10.345442	9.704036	10.295964	10.049478	9.950522	10
51	654808	345192	704350	295650	049542	950458	9
52	655058	344942	704663	295337	049606	950394	8
53	655307	344693	704977	295023	049670	950330	7
54	655556	344444	705290	294710	049734	950266	6
55	9.655805	10.344195	9.705603	10.294397	10.049798	9.950202	5
56	656054	343946	705916	294084	049862	950138	4
57	656302	343698	706228	293772	049926	950074	3
58	656551	343449	706541	293459	049990	950010	2
59	656799	343201	706854	293146	050055	949945	1
60	657047	342953	707166	292834	050119	949881	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

63°

27°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.657047	10.342953	9.707166	10.292834	10.050119	9.949881	60
1	657295	342705	707478	292522	050184	949816	59
2	657542	342458	707790	292210	050248	949752	58
3	657790	342210	708102	291898	050312	949688	57
4	658037	341963	708414	291586	050377	949623	56
5	9.658284	10.341716	9.708726	10.291274	10.050442	9.949558	55
6	658531	341469	709037	290963	050506	949494	54
7	658778	341222	709349	290651	050571	949429	53
8	659025	340975	709660	290340	050636	949364	52
9	659271	340729	709971	290029	050700	949300	51
10	9.659517	10.340483	9.710282	10.289718	10.050765	9.949235	50
11	659763	340237	710593	289407	050830	949170	49
12	660009	339991	710904	289096	050895	949105	48
13	660255	339745	711215	288785	050960	949040	47
14	660501	339499	711525	288475	051025	948975	46
15	9.660746	10.339254	9.711836	10.288164	10.051090	9.948910	45
16	660991	339009	712146	287854	051155	948845	44
17	661236	338764	712456	287544	051220	948780	43
18	661481	338519	712766	287234	051285	948715	42
19	661726	338274	713076	286924	051350	948650	41
20	9.661970	10.338030	9.713386	10.286614	10.051416	9.948584	40
21	662214	337786	713696	286304	051481	948519	39
22	662459	337541	714005	285995	051546	948454	38
23	662703	337297	714314	285686	051612	948388	37
24	662946	337054	714624	285376	051677	948323	36
25	9.663190	10.336810	9.714933	10.285067	10.051743	9.948257	35
26	663433	336567	715242	284758	051808	948192	34
27	663677	336323	715551	284449	051874	948126	33
28	663920	336080	715860	284140	051940	948060	32
29	664163	335837	716168	283832	052005	947995	31
30	9.664406	10.335594	9.716477	10.283523	10.052071	9.947929	30
31	664648	335352	716785	283215	052137	947863	29
32	664891	335109	717093	282907	052203	947797	28
33	665133	334867	717401	282509	052269	947731	27
34	665375	334625	717709	282291	052335	947665	26
35	9.665617	10.334383	9.718017	10.281983	10.052400	9.947600	25
36	665859	334141	718325	281675	052467	947533	24
37	666100	333900	718633	281367	052533	947467	23
38	666342	333658	718940	281060	052599	947401	22
39	666583	333417	719248	280752	052665	947335	21
40	9.666824	10.333176	9.719555	10.280445	10.052731	9.947269	20
41	667065	332935	719862	280138	052797	947203	19
42	667305	332695	720169	279831	052864	947136	18
43	667546·	332454	720476	279524	052930	947070	17
44	667786	332214	720783	279217	052996	947004	16
45	9.668027	10.331973	9.721089	10.278911	10.053063	9.946937	15
46	668267	331733	721396	278604	053129	946871	14
47	668506	331494	721702	278298	053196	946804	13
48	668746	331254	722009	277991	053262	946738	12
49	668986	331014	722315	277685	053329	946671	11
50	9.669225	10.330775	9.722621	10.277379	10.053396	9.946604	10
51	669464	330536	722927	277073	053462	946538	9
52	669703	330297	723232	276768	053529	946471	8
53	669942	330058	723538	276462	053596	946404	7
54	670181	329819	723844	276156	053663	946337	6
55	9.670419	10.329581	9.724149	10.275851	10.053730	9.946270	5
56	670658	329342	724454	275546	053797	946203	4
57	670896	329104	724759	275241	053864	946136	3
58	671134	328866	725065	274935	053931	946069	2
59	671372	328628	725369	274631	053998	946002	1
60	671609	328391	725674	274326	054065	945935	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

62°

28°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.671609	10.328391	9.725674	10.274326	10.054065	9.945935	60
1	671847	328153	725979	274021	054132	945868	59
2	672084	327916	726284	273716	054200	945800	58
3	672321	327679	726588	273412	054267	945733	57
4	672558	327442	726892	273108	054334	945666	56
5	9.672795	10.327205	9.727197	10.272803	10.054402	9.945598	55
6	673032	326968	727501	272499	054469	945531	54
7	673268	326732	727805	272195	054536	945464	53
8	673505	326495	728109	271891	054604	945396	52
9	673741	326259	728412	271588	054672	945328	51
10	9.673977	10.326023	9.728716	10.271284	10.054739	9.945261	50
11	674213	325787	729020	270980	054807	945193	49
12	674448	325552	729323	270677	054875	945125	48
13	674684	325316	729626	270374	054942	945058	47
14	674919	325081	729929	270071	055010	944990	46
15	9.675155	10.324845	9.730233	10.269767	10.055078	9.944922	45
16	675390	324610	730535	269465	055146	944854	44
17	675624	324376	730838	269162	055214	944786	43
18	675859	324141	731141	268859	055282	944718	42
19	676094	323906	731444	268556	055350	944650	41
20	9.676328	10.323672	9.731746	10.268254	10.055418	9.944582	40
21	676562	323438	732048	267952	055486	944514	39
22	676796	323204	732351	267649	055554	944446	38
23	677030	322970	732653	267347	055623	944377	37
24	677264	322736	732955	267045	055691	944309	36
25	9.677498	10.322502	9.733257	10.266743	10.055759	9.944241	35
26	677731	322269	733558	266442	055828	944172	34
27	677964	322036	733860	266140	055896	944104	33
28	678197	321803	734162	265838	055964	944036	32
29	678430	321570	734463	265537	056033	943967	31
30	9.678663	10.321337	9·734764	10.265236	10.056101	9.943899	30
31	678895	321105	735066	264934	056170	943830	29
32	679128	320872	735367	264633	056239	943761	28
33	679360	320640	735668	264332	056307	943693	27
34	679592	320408	735969	264031	056376	943624	26
35	9.679824	10.320176	9.736269	10.263731	10.056445	9.943555	25
36	680056	319944	736570	263430	056514	943486	24
37	680288	319712	736871	263129	056583	943417	23
38	680519	319481	737171	262829	056652	943348	22
39	680750	319250	737471	262529	056721	943279	21
40	9.680982	10.319018	9.737771	10.262229	10.056790	9.943210	20
41	681213	318787	738071	261929	056859	943141	19
42	681443	318557	738371	261629	056928	943072	18
43	681674	318326	738671	261329	056997	943003	17
44	681905	318095	738971	261029	057066	942934	16
45	9.682135	10.317865	9.739271	10.260729	10.057136	9.942864	15
46	682365	317635	739570	260430	057205	942795	14
47	682595	317405	739870	260130	057274	942726	13
48	682825	317175	740169	259831	057344	942656	12
49	683055	316945	740468	259532	057413	942587	11
50	9.683284	10.316716	9.740767	10.259233	10.057483	9.942517	10
51	683514	316486	741066	258934	057552	942448	9
52	683743	316257	741365	258635	057622	942378	8
53	683972	316028	741664	258336	057692	942308	7
54	684201	315799	741962	258038	057761	942239	6
55	9.684430	10.315570	9.742261	10.257739	10.057831	9.942169	5
56	684658	315342	742559	257441	057901	942099	4
57	684887	315113	742858	257142	057971	942029	3
58	685115	314885	743156	256844	058041	941959	2
59	685343	314657	743454	256546	058111	941889	1
60	685571	314429	743752	256248	058181	941819	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

61°

29°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.685571	10.314429	9.743752	10.256248	10.058181	9.941819	60
1	685799	314201	744050	255950	058251	941749	59
2	686027	313973	744348	255652	058321	941679	58
3	686254	313746	744645	255355	058391	941609	57
4	686482	313518	744943	255057	058461	941539	56
5	9.686709	10.313291	9.745240	10.254760	10.058531	9.941469	55
6	686936	313064	745538	254462	058602	941398	54
7	687163	312837	745835	254165	058672	941328	53
8	687389	312611	746132	253868	058742	941258	52
9	687616	312384	746429	253571	058813	941187	51
10	9.687843	10.312157	9.746726	10.253274	10.058883	9.941117	50
11	688069	311931	747023	252977	058954	941046	49
12	688295	311705	747319	252681	059025	940975	48
13	688521	311479	747616	252384	059095	940905	47
14	688747	311253	747913	252087	059166	940834	46
15	9.688972	10.311028	9.748209	10.251791	10.059237	9.940763	45
16	689198	310802	748505	251495	059307	940693	44
17	689423	310577	748801	251199	059378	940622	43
18	689648	310352	749097	250903	059449	940551	42
19	689873	310127	749393	250607	059520	940480	41
20	9.690098	10.309902	9.749689	10.250311	10.059591	9.940409	40
21	690323	309677	749985	250015	059662	940338	39
22	690548	309452	750281	249719	059733	940267	38
23	690772	309228	750576	249424	059804	940196	37
24	690996	309004	750872	249128	059875	940125	36
25	9.691220	10.308780	9.751167	10.548833	10.059946	9.940054	35
26	691444	308556	751462	248538	060018	939982	34
27	691668	308332	751757	248243	060089	939911	33
28	691892	308108	752052	247948	060160	939840	32
29	692115	307885	752347	247653	060232	939768	31
30	9.692339	10.307661	9.752642	10.247358	10.060303	9.939697	30
31	692562	307438	752937	247063	060375	939625	29
32	692785	307215	753231	246769	060446	939554	28
33	693008	306992	753526	246474	060518	939482	27
34	693231	306769	753820	246180	060590	939410	26
35	9.693453	10.306547	9.754115	10.245885	10.060661	9.939339	25
36	693676	306324	754409	245591	060733	939267	24
37	693898	306102	754703	245297	060805	939195	23
38	694120	305880	754997	245003	060877	939123	22
39	694342	305658	755291	244709	060948	939052	21
40	9.694564	10.305436	9.755585	10.244415	10.061020	9.938980	20
41	694786	305214	755878	244122	061092	938908	19
42	695007	304993	756172	243828	061164	938836	18
43	695229	304771	756465	243535	061237	938763	17
44	695450	304550	756759	243241	061309	938691	16
45	9.695671	10.304329	9.757052	10.242948	10.061381	9.938619	15
46	695892	304108	757345	242655	061453	938547	14
47	696113	303887	757638	242362	061525	938475	13
48	696334	303666	757931	242069	061598	938402	12
49	696554	303446	758224	241776	061670	938330	11
50	9.696775	10.303225	9.758517	10.241483	10.061742	9.938258	10
51	696995	303005	758810	241190	061815	938185	9
52	697215	302785	759102	240898	061887	938113	8
53	697435	302565	759395	240605	061960	938040	7
54	697654	302346	759687	240313	062033	937967	6
55	9.697874	10.302126	9.759979	10.240021	10.062105	9.937895	5
56	698094	301906	760272	239728	062178	937822	4
57	698313	301687	760564	239436	062251	937749	3
58	698532	301468	760856	239144	062324	937676	2
59	698751	301249	761148	238852	062396	937604	1
60	698970	301030	761439	238561	062469	937531	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

60°

TABLE III.—*Logarithmic Sines,*

30°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.698970	10.301030	9.761439	10.238561	10.062469	9.937531	60
1	699189	300811	761731	238269	062542	937458	59
2	699407	300593	762023	237977	062615	937385	58
3	699626	300374	762314	237686	062688	937312	57
4	699844	300156	762606	237394	062762	937238	56
5	9.700062	10.299938	9.762897	10.237103	10.062835	9.937165	55
6	700280	299720	763188	236812	062908	937092	54
7	700498	299502	763479	236521	062981	937019	53
8	700716	299284	763770	236230	063054	936946	52
9	700933	299067	764061	235939	063128	936872	51
10	9.701151	10.298849	9.764352	10.235648	10.063201	9.936799	50
11	701368	298632	764643	235357	063275	936725	49
12	701585	298415	764933	235067	063348	936652	48
13	701802	298198	765224	234776	063422	936578	47
14	702019	297981	765514	234486	063495	936505	46
15	9.702236	10.297764	9.765805	10.234195	10.063569	9.936431	45
16	702452	297548	766095	233905	063643	936357	44
17	702669	297331	766385	233615	063716	936284	43
18	702885	297115	766675	233325	063790	936210	42
19	703101	296899	766965	233035	063864	936136	41
20	9.703317	10.296683	9.767255	10.232745	10.063938	9.936062	40
21	703533	296467	767545	232455	064012	935988	39
22	703749	296251	767834	232166	064086	935914	38
23	703964	296036	768124	231876	064160	935840	37
24	704179	295821	768414	231586	064234	935766	36
25	9.704395	10.295605	9.768703	10.231297	10.064308	9.935692	35
26	704610	295390	768992	231008	064382	935618	34
27	704825	295175	769281	230719	064457	935543	33
28	705040	294960	769570	230430	064531	935469	32
29	705254	294746	769860	230140	064605	935395	31
30	9.705469	10.294531	9.770148	10.229852	10.064680	9.935320	30
31	705683	294317	770437	229563	064754	935246	29
32	705898	294102	770726	229274	064829	935171	28
33	706112	293888	771015	228985	064903	935097	27
34	706326	293674	771303	228697	064978	935022	26
35	9.706539	10.293461	9.771592	10.228408	10.065052	9.934948	25
36	706753	293247	771880	228120	065127	934873	24
37	706967	293033	772168	227832	065202	934798	23
38	707180	292820	772457	227543	065277	934723	22
39	707393	292607	772745	227255	065351	934649	21
40	9.707606	10.292394	9.773033	10.226967	10.065426	9.934574	20
41	707819	292181	773321	226679	065501	934499	19
42	708032	291968	773608	226392	065576	934424	18
43	708245	291755	773896	226104	065651	934349	17
44	708458	291542	774184	225816	065726	934274	16
45	9.708670	10.291330	9.774471	10.225529	10.065801	9.934199	15
46	708882	291118	774759	225241	065877	934123	14
47	709094	290906	775046	224954	065952	934048	13
48	709306	290694	775333	224667	066027	933973	12
49	709518	290482	775621	224379	066102	933898	11
50	9.709730	10.290270	9.775908	10.224092	10.066178	9.933822	10
51	709941	290059	776195	223805	066253	933747	9
52	710153	289847	776482	223518	066329	933671	8
53	710364	289636	776769	223231	066404	933596	7
54	710575	289425	777055	222945	066480	933520	6
55	9.710786	10.289214	9.777342	10.222658	10.066555	9.933445	5
56	710997	289003	777628	222372	066631	933369	4
57	711208	288792	777915	222085	066707	933293	3
58	711419	288581	778201	221799	066783	933217	2
59	711629	288371	778488	221512	066859	933141	1
60	711839	288161	778774	221226	066934	933066	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

59°

31°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.711839	10.288161	9.778774	10.221226	10.066934	9.933066	60
1	712050	287950	779060	220940	067010	932990	59
2	712260	287740	779346	220654	067086	932914	58
3	712469	287531	779632	220368	067162	932838	57
4	712679	287321	779918	220082	067238	932762	56
5	9.712889	10.287111	9.780203	10.219797	10.067315	9.932685	55
6	713098	286902	780489	219511	067391	932609	54
7	713308	286692	780775	219225	067467	932533	53
8	713517	286483	781060	218940	067543	932457	52
9	713726	286274	781346	218654	067620	932380	51
10	9.713935	10.286065	9.781631	10.218369	10.067696	9.932304	50
11	714144	285856	781916	218084	067772	932228	49
12	714352	285648	782201	217799	067849	932151	48
13	714561	285439	782486	217514	067925	932075	47
14	714769	285231	782771	217229	068002	931998	46
15	9.714978	10.285022	9.783056	10.216944	10.068079	9.931921	45
16	715186	284814	783341	216659	068155	931845	44
17	715394	284606	783626	216374	068232	931768	43
18	715602	284398	783910	216090	068309	931691	42
19	715809	284191	784195	215805	068386	931614	41
20	9.716017	10.283983	9.784479	10.215521	10.068463	9.931537	40
21	716224	283776	784764	215236	068540	931460	39
22	716432	283568	785048	214952	068617	931383	38
23	716639	283361	785332	214668	068694	931306	37
24	716846	283154	785616	214384	068771	931229	36
25	9.717053	10.282947	9.785900	10.214100	10.068848	9.931152	35
26	717259	282741	786184	213816	068925	931075	34
27	717466	282534	786468	213532	069002	930998	33
28	717673	282327	786752	213248	069079	930921	32
29	717879	282121	787036	212964	069157	930843	31
30	9.718085	10.281915	9.787319	10.212681	10.069234	9.930766	30
31	718291	281709	787603	212397	069312	930688	29
32	718497	281503	787886	212114	069389	930611	28
33	718703	281297	788170	211830	069467	930533	27
34	718909	281091	788453	211547	069544	930456	26
35	9.719114	10.280886	9.788736	10.211264	10.069622	9.930378	25
36	719320	280680	789019	210981	069700	930300	24
37	719525	280475	789302	210698	069777	930223	23
38	719730	280270	789585	210415	069855	930145	22
39	719935	280065	789868	210132	069933	930067	21
40	9.720140	10.279860	9.790151	10.209849	10.070011	9.929989	20
41	720345	279655	790433	209567	070089	929911	19
42	720549	279451	790716	209284	070167	929833	18
43	720754	279246	790999	209001	070245	929755	17
44	720958	279042	791281	208719	070323	929677	16
45	9.721162	10.278838	9.791563	10.208437	10.070401	9.929599	15
46	721366	278634	791846	208154	070479	929521	14
47	721570	278430	792128	207872	070558	929442	13
48	721774	278226	792410	207590	070636	929364	12
49	721978	278022	792692	207308	070714	929286	11
50	9.722181	10.277819	9.792974	10.207026	10.070793	9.929207	10
51	722385	277615	793256	206744	070871	929129	9
52	722588	277412	793538	206462	070950	929050	8
53	722791	277209	793819	206181	071028	928972	7
54	722994	277006	794101	205899	071107	928893	6
55	9.723197	10.276803	9.794383	10.205617	10.071185	9.928815	5
56	723400	276600	794664	205336	071264	928736	4
57	723603	276397	794945	205055	071343	928657	3
58	723805	276195	795227	204773	071422	928578	2
59	724007	275993	795508	204492	071501	928499	1
60	724210	275790	795789	204211	071580	928420	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

58°

32°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.724210	10.275790	9.795789	10.204211	10.071580	9.928420	60
1	724412	275588	796070	203930	071658	928342	59
2	724614	275386	796351	203649	071737	928263	58
3	724816	275184	796632	203368	071817	928183	57
4	725017	274983	796913	203087	071896	928104	56
5	9.725219	10.274781	9.797194	10.202806	10.071975	9.928025	55
6	725420	274580	797475	202525	072054	927946	54
7	725622	274378	797755	202245	072133	927867	53
8	725823	274177	798036	201964	072213	927787	52
9	726024	273976	798316	201684	072292	927708	51
10	9.726225	10.273775	9.798596	10.201404	10.072371	9.927629	50
11	726426	273574	798877	201123	072451	927549	49
12	726626	273374	799157	200843	072530	927470	48
13	726827	273173	799437	200563	072610	927390	47
14	727027	272973	799717	200283	072690	927310	46
15	9.727228	10.272772	9.799997	10.200003	10.072769	9.927231	45
16	727428	272572	800277	199723	072849	927151	44
17	727628	272372	800557	199443	072929	927071	43
18	727828	272172	800836	199164	073009	926991	42
19	728027	271973	801116	198884	073089	926911	41
20	9.728227	10.271773	9.801396	10.198604	10.073169	9.926831	40
21	728427	271573	801675	198325	073249	926751	39
22	728626	271374	801955	198045	073329	926671	38
23	728825	271175	802234	197766	073409	926591	37
24	729024	270976	802513	197487	073489	926511	36
25	9.729223	10.270777	9.802792	10.197208	10.073569	9.926431	35
26	729422	270578	803072	196928	073649	926351	34
27	729621	270379	803351	196649	073730	926270	33
28	729820	270180	803630	196370	073810	926190	32
29	730018	269982	803908	196092	073890	926110	31
30	9.730217	10.269783	9.804187	10.195813	10.073971	9.926029	30
31	730415	269585	804466	195534	074051	925949	29
32	730613	269387	804745	195255	074132	925868	28
33	730811	269189	805023	194977	074212	925788	27
34	731009	268991	805302	194698	074293	925707	26
35	9.731206	10.268794	9.805580	10.194420	10.074374	9.925626	25
36	731404	268596	805859	194141	074455	925545	24
37	731602	268398	806137	193863	074535	925465	23
38	731799	268201	806415	193585	074616	925384	22
39	731996	268004	806693	193307	074697	925303	21
40	9.732193	10.267807	9.806971	10.193029	10.074778	9.925222	20
41	732390	267610	807249	192751	074859	925141	19
42	732587	267413	807527	192473	074940	925060	18
43	732784	267216	807805	192195	075021	924979	17
44	732980	267020	808083	191917	075103	924897	16
45	9.733177	10.266823	9.808361	10.191639	10.075184	9.924816	15
46	733373	266627	808638	191362	075265	924735	14
47	733569	266431	808916	191084	075346	924654	13
48	733765	266235	809193	190807	075428	924572	12
49	733961	266039	809471	190529	075509	924491	11
50	9.734157	10.265843	9.809748	10.190252	10.075591	9.924409	10
51	734353	265647	810025	189975	075672	924328	9
52	734549	265451	810302	189698	075754	924246	8
53	734744	265256	810580	189420	075836	924164	7
54	734939	265061	810857	189143	075917	924083	6
55	9.735135	10.264865	9.811134	10.188866	10.075999	9.924001	5
56	735330	264670	811410	188590	076081	923919	4
57	735525	264475	811687	188313	076163	923837	3
58	735719	264281	811964	188036	076245	923755	2
59	735914	264086	812241	187759	076327	923673	1
60	736109	263891	812517	187483	076409	923591	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

57°

33°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.736109	10.263891	9.812517	10.187483	10.076409	9.923591	60
1	736303	263697	812794	187206	076491	923509	59
2	736498	263502	813070	186930	076573	923427	58
3	736692	263308	813347	186653	076655	923345	57
4	736886	263114	813623	186377	076737	923263	56
5	9.737080	10.262920	9.813899	10.186101	10.076819	9.923181	55
6	737274	262726	814175	185825	076902	923098	54
7	737467	262533	814452	185548	076984	923016	53
8	737661	262339	814728	185272	077067	922933	52
9	737855	262145	815004	184996	077149	922851	51
10	9.738048	10.261952	9.815279	10.184721	10.077232	9.922768	50
11	738241	261759	815555	184445	077314	922686	49
12	738434	261566	815831	184169	077397	922603	48
13	738627	261373	816107	183893	077480	922520	47
14	738820	261180	816382	183618	077562	922438	46
15	9.739013	10.260987	9.816658	10.183342	10.077645	9.922355	45
16	739206	260794	816933	183067	077728	922272	44
17	739398	260602	817209	182791	077811	922189	43
18	739590	260410	817484	182516	077894	922106	42
19	739783	260217	817759	182241	077977	922023	41
20	9.739975	10.260025	9.818035	10.181965	10.078060	9.921940	40
21	740167	259833	818310	181690	078143	921857	39
22	740359	259641	818585	181415	078226	921774	38
23	740550	259450	818860	181140	078309	921691	37
24	740742	259258	819135	180865	078393	921607	36
25	9.740934	10.259066	9.819410	10.180590	10.078476	9.921524	35
26	741125	258875	819684	180316	078559	921441	34
27	741316	258684	819959	180041	078643	921357	33
28	741508	258492	820234	179766	078726	921274	32
29	741699	258301	820508	179492	078810	921190	31
30	9.741889	10.258111	9·820783	10.179217	10.078893	9.921107	30
31	742080	257920	821057	178943	078977	921023	29
32	742271	257729	821332	178668	079061	920939	28
33	742462	257538	821606	178394	079144	920856	27
34	742652	257348	821880	178120	079228	920772	26
35	9.742842	10.257158	9.822154	10.177846	10.079312	9.920688	25
36	743033	256967	822429	177571	079396	920604	24
37	743223	256777	822703	177297	079480	920520	23
38	743413	256587	822977	177023	079564	920436	22
39	743602	256398	823250	176750	079648	920352	21
40	9.743792	10.256208	9.823524	10.176476	10.079732	9.920268	20
41	743982	256018	823798	176202	079816	920184	19
42	744171	255829	824072	175928	079901	920099	18
43	744361	255639	824345	175655	079985	920015	17
44	744550	255450	824619	175381	080069	919931	16
45	9.744739	10.255261	9.824893	10.175107	10.080154	9.919846	15
46	744928	255072	825166	174834	080238	919762	14
47	745117	254883	825439	174561	080323	919677	13
48	745306	254694	825713	174287	080407	919593	12
49	745494	254506	825986	174014	080492	919508	11
50	9.745683	10.254317	9.826259	10.173741	10.080576	9.919424	10
51	745871	254129	826532	173468	080661	919339	9
52	746059	253941	826805	173195	080746	919254	8
53	746248	253752	827078	172922	080831	919169	7
54	746436	253564	827351	172649	080915	919085	6
55	9.746624	10.253376	9.827624	10.172376	10.081000	9.919000	5
56	746812	253188	827897	172103	081085	918915	4
57	746999	253001	828170	171830	081170	918830	3
58	747187	252813	828442	171558	081255	918745	2
59	747374	252626	828715	171285	081341	918659	1
60	747562	252438	828987	171013	081426	918574	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

56°

TABLE III.—*Logarithmic Sines,*

34°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.747562	10.252438	9.828987	10.171013	10.081426	9.918574	60
1	747749	252251	829260	170740	081511	918489	59
2	747936	252064	829532	170468	081596	918404	58
3	748123	251877	829805	170195	081682	918318	57
4	748310	251690	830077	169923	081767	918233	56
5	9.748497	10.251503	9.830349	10.169651	10.081853	9.918147	55
6	748683	251317	830621	169379	081938	918062	54
7	748870	251130	830893	169107	082024	917976	53
8	749056	250944	831165	168835	082109	917891	52
9	749243	250757	831437	168563	082195	917805	51
10	9.749429	10.250571	9.831709	10.168291	10.082281	9.917719	50
11	749615	250385	831981	168019	082366	917634	49
12	749801	250199	832253	167747	082452	917548	48
13	749987	250013	832525	167475	082538	917462	47
14	750172	249828	832796	167204	082624	917376	46
15	9.750358	10.249642	9.833068	10.166932	10.082710	9.917290	45
16	750543	249457	833339	166661	082796	917204	44
17	750729	249271	833611	166389	082882	917118	43
18	750914	249086	833882	166118	082968	917032	42
19	751099	248901	834154	165846	083054	916946	41
20	9.751284	10.248716	9.834425	10.165575	10.083141	9.916859	40
21	751469	248531	834696	165304	083227	916773	39
22	751654	248346	834967	165033	083313	916687	38
23	751839	248161	835238	164762	083400	916600	37
24	752023	247977	835509	164491	083486	916514	36
25	9.752208	10.247792	9.835780	10.164220	10.083573	9.916427	35
26	752392	247608	836051	163949	083659	916341	34
27	752576	247424	856322	163678	083746	916254	33
28	752760	247240	836593	163407	083833	916167	32
29	752944	247056	836864	163136	083919	916081	31
30	9.753128	10.246872	9.837134	10.162866	10.084006	9.915994	30
31	753312	246688	837405	162595	084093	915907	29
32	753495	246505	837675	162325	084180	915820	28
33	753679	246321	837946	162054	084267	915733	27
34	753862	246138	838216	161784	084354	915646	26
35	9.754046	10.245954	9.838487	10.161513	10.084441	9.915559	25
36	754229	245771	838757	161243	084528	915472	24
37	754412	245588	839027	160973	084615	915385	23
38	754595	245405	839297	160703	084703	915297	22
39	754778	245222	839568	160432	084790	915210	21
40	9.754960	10.245040	9.839838	10.160162	10.084877	9.915123	20
41	755143	244857	840108	159892	084965	915035	19
42	755326	244674	840378	159622	085052	914948	18
43	755508	244492	840647	159353	085140	914860	17
44	755690	244310	840917	159083	085227	914773	16
45	9.755872	10.244128	9.841187	10.158813	10.085315	9.914685	15
46	756054	243946	841457	158543	085402	914598	14
47	756236	243764	841726	158274	085490	914510	13
48	756418	243582	841996	158004	085578	914422	12
49	756600	243400	842266	157734	085666	914334	11
50	9.756782	10.243218	9.842535	10.157465	10.085754	9.914246	10
51	756963	243037	842805	157195	085842	914158	9
52	757144	242856	843074	156926	085930	914070	8
53	757326	242674	843343	156657	086018	913982	7
54	757507	242493	843612	156388	086106	913894	6
55	9.757688	10.242312	9.843882	10.156118	10.086194	9.913806	5
56	757869	242131	844151	155849	086282	913718	4
57	758050	241950	844420	155580	086370	913630	3
58	758230	241770	844689	155311	086459	913541	2
59	758411	241589	844958	155042	086547	913453	1
60	758591	241409	845227	154773	086635	913365	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

55°

35°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.758591	10.241409	9.845227	10.154773	10.086635	9.913365	60
1	758772	241228	845496	154504	086724	913276	59
2	758952	241048	845764	154236	086813	913187	58
3	759132	240868	846033	153967	086901	913099	57
4	759312	240688	846302	153698	086990	913010	56
5	9.759492	10.240508	9.846570	10.153430	10.087078	9.912922	55
6	759672	240328	846839	153161	087167	912833	54
7	759852	240148	847107	152893	087256	912744	53
8	760031	239969	847376	152624	087345	912655	52
9	760211	239789	847644	152356	087434	912566	51
10	9.760390	10.239610	9.847913	10.152087	10.087523	9.912477	50
11	760569	239431	848181	151819	087612	912388	49
12	760748	239252	848449	151551	087701	912299	48
13	760927	239073	848717	151283	087790	912210	47
14	761106	238894	848986	151014	087879	912121	46
15	9.761285	10.238715	9.849254	10.150746	10.087969	9.912031	45
16	761464	238536	849522	150478	088058	911942	44
17	761642	238358	849790	150210	088147	911853	43
18	761821	238179	850058	149942	088237	911763	42
19	761999	238001	850325	149675	088326	911674	41
20	9.762177	10.237823	9.850593	10.149407	10.088416	9.911584	40
21	762356	237644	850861	149139	088505	911495	39
22	762534	237466	851129	148871	088595	911405	38
23	762712	237288	851396	148604	088685	911315	37
24	762889	237111	851664	148336	088774	911226	36
25	9.763067	10.236933	9.851931	10.148069	10.088864	9.911136	35
26	763245	236755	852199	147801	088954	911046	34
27	763422	236578	852466	147534	089044	910956	33
28	763600	236400	852733	147267	089134	910866	32
29	763777	236223	853001	146999	089224	910776	31
30	9.763954	10.236046	9.853268	10.146732	10.089314	9.910686	30
31	764131	235869	853535	146465	089404	910596	29
32	764308	235692	853802	146198	089494	910506	28
33	764485	235515	854069	145931	089585	910415	27
34	764662	235338	854336	145664	089675	910325	26
35	9.764838	10.235162	9.854603	10.145397	10.089765	9.910235	25
36	765015	234985	854870	145130	089856	910144	24
37	765191	234809	855137	144863	089946	910054	23
38	765367	234633	855404	144596	090037	909963	22
39	765544	234456	855671	144329	090127	909873	21
40	9.765720	10.234280	9.855938	10.144062	10.090218	9.909782	20
41	765896	234104	856204	143796	090309	909691	19
42	766072	233928	856471	143529	090399	909601	18
43	766247	233753	856737	143263	090490	909510	17
44	766423	233577	857004	142996	090581	909419	16
45	9.766598	10.233402	9.857270	10.142730	10.090672	9.909328	15
46	766774	233226	857537	142463	090763	909237	14
47	766949	233051	857803	142197	090854	909146	13
48	767124	232876	858069	141931	090945	909055	12
49	767300	232700	858336	141664	091036	908964	11
50	9.767475	10.232525	9.858602	10.141398	10.091127	9.908873	10
51	767649	232351	858868	141132	091219	908781	9
52	767824	232176	859134	140866	091310	908690	8
53	767999	232001	859400	140600	091401	908599	7
54	768173	231827	859666	140334	091493	908507	6
55	9.768348	10.231652	9.859932	10.140068	10.091584	9.908416	5
56	768522	231478	860198	139802	091676	908324	4
57	768697	231303	860464	139536	091767	908233	3
58	768871	231129	860730	139270	091859	908141	2
59	769045	230955	860995	139005	091951	908049	1
60	769219	230781	861261	138739	092042	907958	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

54°

TABLE III.—*Logarithmic Sines,*

36°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.769219	10.230781	9.861261	10.138739	10.092042	9.907958	60
1	769393	230607	861527	138473	092134	907866	59
2	769566	230434	861792	138208	092226	907774	58
3	769740	230260	862058	137942	092318	907682	57
4	769913	230087	862323	137677	092410	907590	56
5	9.770087	10.229913	9.862589	10.137411	10.092502	9.907498	55
6	770260	229740	862854	137146	092594	907406	54
7	770433	229567	863119	136881	092686	907314	53
8	770606	229394	863385	136615	092778	907222	52
9	770779	229221	863650	136350	092871	907129	51
10	9.770952	10.229048	9.863915	10.136085	10.092963	9.907037	50
11	771125	228875	864180	135820	093055	906945	49
12	771298	228702	864445	135555	093148	906852	48
13	771470	228530	864710	135290	093240	906760	47
14	771643	228357	864975	135025	093333	906667	46
15	9.771815	10.228185	9.865240	10.134760	10.093425	9.906575	45
16	771987	228013	865505	134495	093518	906482	44
17	772159	227841	865770	134230	093611	906389	43
18	772331	227669	866035	133965	093704	906296	42
19	772503	227497	866300	133700	093796	906204	41
20	9.772675	10.227325	9.866564	10.133436	10.093889	9.906111	40
21	772847	227153	866829	133171	093982	906018	39
22	773018	226982	8670..4	132906	094075	905925	38
23	773190	226810	867358	132642	094168	905832	37
24	773361	226639	867623	132377	094261	905739	36
25	9.773533	10.226467	9.867887	10.132113	10.094355	9.905645	35
26	773704	226296	868152	131848	094448	905552	34
27	773875	226125	868416	131584	094541	905459	33
28	774046	225954	868680	131320	094634	905366	32
29	774217	225783	868945	131055	094728	905272	31
30	9.774388	10.225612	9.869209	10.130791	10.094821	9.905179	30
31	774558	225442	869473	130527	094915	905085	29
32	774729	225271	869737	130263	095008	904992	28
33	774899	225101	870001	129999	095102	904898	27
34	775070	224930	870265	129735	095196	904804	26
35	9.775240	10.224760	9.870529	10.129471	10.095289	9.904711	25
36	775410	224590	870793	129207	095383	904617	24
37	775580	224420	871057	128943	095477	904523	23
38	775750	224250	871321	128679	095571	904429	22
39	775920	224080	871585	128415	095665	904335	21
40	9.776090	10.223910	9.871849	10.128151	10.095759	9.904241	20
41	776259	223741	872112	127888	095853	904147	19
42	776429	223571	872376	127624	095947	904053	18
43	776598	223402	872640	127360	096041	903959	17
44	776768	223232	872903	127097	096136	903864	16
45	9.776937	10.223063	9.873167	10.126833	10.096230	9.903770	15
46	777106	222894	873430	126570	096324	903676	14
47	777275	222725	873694	126306	096419	903581	13
48	777444	222556	873957	126043	096513	903487	12
49	777613	222387	874220	125780	096608	903392	11
50	9.777781	10.222219	9.874484	10.125516	10.096702	9.903298	10
51	777950	222050	874747	125253	096797	903203	9
52	778119	221881	875010	124990	096892	903108	8
53	778287	221713	875273	124727	096986	903014	7
54	778455	221545	875536	124464	097081	902919	6
55	9.778624	10.221376	9.875800	10.124200	10.097176	9.902824	5
56	778792	221208	876063	123937	097271	902729	4
57	778960	221040	876326	123674	097366	902634	3
58	779128	220872	876589	123411	097461	902539	2
59	779295	220705	876851	123149	097556	902444	1
60	779463	220537	877114	122886	097651	902349	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

53°

37°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.779463	10.220537	9.877114	10.122886	10.097651	9.902349	60
1	779631	220369	877377	122623	097747	902253	59
2	779798	220202	877640	122360	097842	902158	58
3	779966	220034	877903	122097	097937	902063	57
4	780133	219867	878165	121835	098033	901967	56
5	9.780300	10.219700	9.878428	10.121572	10.098128	9.901872	55
6	780467	219533	878691	121309	098224	901776	54
7	780634	219366	878953	121047	098319	901681	53
8	780801	219199	879216	120784	098415	901585	52
9	780968	219032	879478	120522	098510	901490	51
10	9.781134	10.218866	9.879741	10.120259	10.098606	9.901394	50
11	781301	218699	880003	119997	098702	901298	49
12	781468	218532	880265	119735	098798	901202	48
13	781634	218366	880528	119472	098894	901106	47
14	781800	218200	880790	119210	098990	901010	46
15	9.781966	10.218034	9.881052	10.118948	10.099086	9.900914	45
16	782132	217868	881314	118686	099182	900818	44
17	782298	217702	881576	118424	099278	900722	43
18	782464	217536	881839	118161	099374	900626	42
19	782630	217370	882101	117899	099471	900529	41
20	9.782796	10.217204	9.882363	10.117637	10.099567	9.900433	40
21	782961	217039	882625	117375	099663	900337	39
22	783127	216873	882887	117113	099760	900240	38
23	783292	216708	883148	116852	099856	900144	37
24	783458	216542	883410	116590	099953	900047	36
25	9.783623	10.216377	9.883672	10.116328	10.100049	9.899951	35
26	733788	216212	883934	116066	100146	899854	34
27	783953	216047	884196	115804	100243	899757	33
28	784118	215882	884457	115543	100340	899660	32
29	784282	215718	884719	115281	100436	899564	31
30	9.784447	10.215553	9.884980	10.115020	10.100533	9.899467	30
31	784612	215388	885242	114758	100630	899370	29
32	784776	215224	885503	114497	100727	899273	28
33	784941	215059	885765	114235	100824	899176	27
34	785105	214895	886026	113974	100922	899078	26
35	9.785269	10.214731	9.886288	10.113712	10.101019	9.898981	25
36	785433	214567	886549	113451	101116	898884	24
37	785597	214403	886810	113190	101213	898787	23
38	785761	214239	887072	112928	101311	898689	22
39	785925	214075	887333	112667	101408	898592	21
40	9.786089	10.213911	9.887594	10.112406	10.101506	9.898494	20
41	786252	213748	887855	112145	101603	898397	19
42	786416	213584	888116	111884	101701	898299	18
43	786579	213421	888377	111623	101798	898202	17
44	786742	213258	888639	111361	101896	898104	16
45	9.786906	10.213094	9.888900	10.111100	10.101994	9.898006	15
46	787069	212931	889160	110840	102092	897908	14
47	787232	212768	889421	110579	102190	897810	13
48	787395	212605	889682	110318	102288	897712	12
49	787557	212443	889943	110057	102386	897614	11
50	9.787720	10.212280	9.890204	10.109796	10.102484	9.897516	10
51	787883	212117	890465	109535	102582	897418	9
52	788045	211955	890725	109275	102680	897320	8
53	788208	211792	890986	109014	102778	897222	7
54	788370	211630	891247	108753	102877	897123	6
55	9.788532	10.211468	9.891507	10.108493	10.102975	9.897025	5
56	788694	211306	891768	108232	103074	896926	4
57	788856	211144	892028	107972	103172	896828	3
58	789018	210982	892289	107711	103271	896729	2
59	789180	210820	892549	107451	103369	896631	1
60	789342	210658	892810	107190	103468	896532	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

52°

38°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.789342	10.210658	9.892810	10.107190	10.103468	9.896532	60
1	789504	210496	893070	106930	103567	896433	59
2	789665	210335	893331	106669	103665	896335	58
3	789827	210173	893591	106409	103764	896236	57
4	789988	210012	893851	106149	103863	896137	56
5	9.790149	10.209851	9.894111	10.105889	10.103962	9.896038	55
6	790310	209690	894371	105629	104061	895939	54
7	790471	209529	894632	105368	104160	895840	53
8	790632	209368	894892	105108	104259	895741	52
9	790793	209207	895152	104848	104359	895641	51
10	9.790954	10.209046	9.895412	10.104588	10.104458	9.895542	50
11	791115	208885	895672	104328	104557	895443	49
12	791275	208725	895932	104068	104657	895343	48
13	791436	208564	896192	103808	104756	895244	47
14	791596	208404	896452	103548	104855	895145	46
15	9.791757	10.208243	9.896712	10.103288	10.104955	9.895045	45
16	791917	208083	896971	103029	105055	894945	44
17	792077	207923	897231	102769	105154	894846	43
18	792237	207763	897491	102509	105254	894746	42
19	792397	207603	897751	102249	105354	894646	41
20	9.792557	10.207443	9.898010	10.101990	10.105454	9.894546	40
21	792716	207284	898270	101730	105554	894446	39
22	792876	207124	898530	101470	105654	894346	38
23	793035	206965	898789	101211	105754	894246	37
24	793195	206805	899049	100951	105854	894146	36
25	9.793354	10.206646	9.899308	10.100692	10.105954	9.894046	35
26	793514	206486	899568	100432	106054	893946	34
27	793673	206327	899827	100173	106154	893846	33
28	793832	206168	900086	099914	106255	893745	32
29	793991	206009	900346	099654	106355	893645	31
30	9.794150	10.205850	9·900605	10.099395	10.106456	9.893544	30
31	794308	205692	900864	099136	106556	893444	29
32	794467	205533	901124	098876	106657	893343	28
33	794626	205374	901383	098617	106757	893243	27
34	794784	205216	901642	098358	106858	893142	26
35	9.794942	10.205058	9.901901	10.098099	10.106959	9.893041	25
36	795101	204899	902160	097840	107060	892940	24
37	795259	204741	902419	097581	107161	892839	23
38	795417	204583	902679	097321	107261	892739	22
39	795575	204425	902938	097062	107362	892638	21
40	9.795733	10.204267	9.903197	10.096803	10.107464	9.892536	20
41	795891	204109	903455	096545	107565	892435	19
42	796049	203951	903714	096286	107666	892334	18
43	796206	203794	903973	096027	107767	892233	17
44	796364	203636	904232	095768	107868	892132	16
45	9.796521	10.203479	9.904491	10.095509	10.107970	9.892030	15
46	796679	203321	904750	095250	108071	891929	14
47	796836	203164	905008	094992	108173	891827	13
48	796993	203007	905267	094733	108274	891726	12
49	797150	202850	905526	094474	108376	891624	11
50	9.797307	10.202693	9.905784	10.094216	10.108477	9.891523	10
51	797464	202536	906043	093957	108579	891421	9
52	797621	202379	906302	093698	108681	891319	8
53	797777	202223	906560	093440	108783	891217	7
54	797934	202066	906819	093181	108885	891115	6
55	9.798091	10.201909	9.907077	10.092923	10.108987	9.891013	5
56	798247	201753	907336	092664	109089	890911	4
57	798403	201597	907594	092406	109191	890809	3
58	798560	201440	907852	092148	109293	890707	2
59	798716	201284	908111	091889	109395	890605	1
60	798872	201128	908369	091631	109497	890503	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

51°

39°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.798872	10.201128	9.908369	10.091631	10.109497	9.890503	60
1	799028	200972	908628	091372	109600	890400	59
2	799184	200816	908886	091114	109702	890298	58
3	799339	200661	909144	090856	109805	890195	57
4	799495	200505	909402	090598	109907	890093	56
5	9.799651	10.200349	9.909660	10.090340	10.110010	9.889990	55
6	799806	200194	909918	090082	110112	889888	54
7	799962	200038	910177	089823	110215	889785	53
8	800117	199883	910435	089565	110318	889682	52
9	800272	199728	910693	089307	110421	889579	51
10	9.800427	10.199573	9.910951	10.089049	10.110523	9.889477	50
11	800582	199418	911209	088791	110626	889374	49
12	800737	199263	911467	088533	110729	889271	48
13	800892	199108	911724	088276	110832	889168	47
14	801047	198953	911982	088018	110936	889064	46
15	9.801201	10.198799	9.912240	10.087760	10.111039	9.888961	45
16	801356	198644	912498	087502	111142	888858	44
17	801511	198489	912756	087244	111245	888755	43
18	801665	198335	913014	086986	111349	888651	42
19	801819	198181	913271	086729	111452	888548	41
20	9.801973	10.198027	9.913529	10.086471	10.111556	9.888444	40
21	802128	197872	913787	086213	111659	888341	39
22	802282	197718	914044	085956	111763	888237	38
23	802436	197564	914302	085698	111866	888134	37
24	802589	197411	914560	085440	111970	888030	36
25	9.802743	10.197257	9.914817	10.085183	10.112074	9.887926	35
26	802897	197103	915075	084925	112178	887822	34
27	803050	196950	915332	084668	112282	887718	33
28	803204	196796	915590	084410	112386	887614	32
29	803357	196643	915847	084153	112490	887510	31
30	9.803511	10.196489	9.916104	10.083896	10.112594	9.887406	30
31	803664	196336	916362	083638	112698	887302	29
32	803817	196183	916619	083381	112802	887198	28
33	803970	196030	916877	083123	112907	887093	27
34	804123	195877	917134	082866	113011	886989	26
35	9.804276	10.195724	9.917391	10.082609	10.113115	9.886885	25
36	804428	195572	917648	082352	113220	886780	24
37	804581	195419	917905	082095	113324	886676	23
38	804734	195266	918163	081837	113429	886571	22
39	804886	195114	918420	081580	113534	886466	21
40	9.805039	10.194961	9.918677	10.081323	10.113638	9.886362	20
41	805191	194809	918934	081066	113743	886257	19
42	805343	194657	919191	080809	113848	886152	18
43	805495	194505	919448	080552	113953	886047	17
44	805647	194353	919705	080295	114058	885942	16
45	9.805799	10.194201	9.919962	10.080038	10.114163	9.885837	15
46	805951	194049	920219	079781	114268	885732	14
47	806103	193897	920476	079524	114373	885627	13
48	806254	193746	920733	079267	114478	885522	12
49	806406	193594	920990	079010	114584	885416	11
50	9.806557	10.193443	9.921247	10.078753	10.114689	9.885311	10
51	806709	193291	921503	078497	114795	885205	9
52	806860	193140	921760	078240	114900	885100	8
53	807011	192989	922017	077983	115006	884994	7
54	807163	192837	922274	077726	115111	884889	6
55	9.807314	10.192686	9.922530	10.077470	10.115217	9.884783	5
56	807465	192535	922787	077213	115323	884677	4
57	807615	192385	923044	076956	115428	884572	3
58	807766	192234	923300	076700	115534	884466	2
59	807917	192083	923557	076443	115640	884360	1
60	808067	191933	923813	076187	115746	884254	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

50°

40°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.808067	10.191933	9.923813	10.076187	10.115746	9.884254	60
1	808218	191782	924070	075930	115852	884148	59
2	808368	191632	924327	075673	115958	884042	58
3	808519	191481	924583	075417	116064	883936	57
4	808669	191331	924840	075160	116171	883829	56
5	9.808819	10.191181	9.925096	10.074904	10.116277	9.883723	55
6	808969	191031	925352	074648	116383	883617	54
7	809119	190881	925609	074391	116490	883510	53
8	809269	190731	925865	074135	116596	883404	52
9	809419	190581	926122	073878	116703	883297	51
10	9.809569	10.190431	9.926378	10.073622	10.116809	9.883191	50
11	809718	190282	926634	073366	116916	883084	49
12	809868	190132	926890	073110	117023	882977	48
13	810017	189983	927147	072853	117129	882871	47
14	810167	189833	927403	072597	117236	882764	46
15	9.810316	10.189684	9.927659	10.072341	10.117343	9.882657	45
16	810465	189535	927915	072085	117450	882550	44
17	810614	189386	928171	071829	117557	882443	43
18	810763	189237	928427	071573	117664	882336	42
19	810912	189088	928683	071317	117771	882229	41
20	9.811061	10.188939	9.928940	10.071060	10.117879	9.882121	40
21	811210	188790	929196	070804	117986	882014	39
22	811358	188642	929452	070548	118093	881907	38
23	811507	188493	929708	070292	118201	881799	37
24	811655	188345	929964	070036	118308	881692	36
25	9.811804	10.188196	9.930220	10.069780	10.118416	9.881584	35
26	811952	188048	930475	069525	118523	881477	34
27	812100	187900	930731	069269	118631	881369	33
28	812248	187752	930987	069013	118739	881261	32
29	812396	187604	931243	068757	118847	881153	31
30	9.812544	10.187456	9.931499	10.068501	10.118954	9.881046	30
31	812692	187308	931755	068245	119062	880938	29
32	812840	187160	932010	067990	119170	880830	28
33	812988	187012	932266	067734	119278	880722	27
34	813135	186865	932522	067478	119387	880613	26
35	9.813283	10.186717	9.932778	10.067222	10.119495	9.880505	25
36	813430	186570	933033	066967	119603	880397	24
37	813578	186422	933289	066711	119711	880289	23
38	813725	186275	933545	066455	119820	880180	22
39	813872	186128	933800	066200	119928	880072	21
40	9.814019	10.185981	9.934056	10.065944	10.120037	9.879963	20
41	814166	185834	934311	065689	120145	879855	19
42	814313	185687	934567	065433	120254	879746	18
43	814460	185540	934823	065177	120363	879637	17
44	814607	185393	935078	064922	120471	879529	16
45	9.814753	10.185247	9.935333	10.064667	10.120580	9.879420	15
46	814900	185100	935589	064411	120689	879311	14
47	815046	184954	935844	064156	120798	879202	13
48	815193	184807	936100	063900	120907	879093	12
49	815339	184661	936355	063645	121016	878984	11
50	9.815485	10.184515	9.936610	10.063390	10.121125	9.878875	10
51	815632	184368	936866	063134	121234	878766	9
52	815778	184222	937121	062879	121344	878656	8
53	815924	184076	937376	062624	121453	878547	7
54	816069	183931	937632	062368	121562	878438	6
55	9.816215	10.183785	9.937887	10.062113	10.121672	9.878328	5
56	816361	183639	938142	061858	121781	878219	4
57	816507	183493	938398	061602	121891	878109	3
58	816652	183348	938653	061347	122001	877999	2
59	816798	183202	938908	061092	122110	877890	1
60	816943	183057	939163	060837	122220	877780	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

49°

41°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.816943	10.183057	9.939163	10.060837	10.122220	9.877780	60
1	817088	182912	939418	060582	122330	877670	59
2	817233	182767	939673	060327	122440	877560	58
3	817379	182621	939928	060072	122550	877450	57
4	817524	182476	940183	059817	122660	877340	56
5	9.817668	10.182332	9.940438	10.059562	10.122770	9.877230	55
6	817813	182187	940694	059306	122880	877120	54
7	817958	182042	940949	059051	122990	877010	53
8	818103	181897	941204	058796	123101	876899	52
9	818247	181753	941458	058542	123211	876789	51
10	9.818392	10.181608	9.941714	10.058286	10.123322	9.876678	50
11	818536	181464	941968	058032	123432	876568	49
12	818681	181319	942223	057777	123543	876457	48
13	818825	181175	942478	057522	123653	876347	47
14	818969	181031	942733	057267	123764	876236	46
15	9.819113	10.180887	9.942988	10.057012	10.123875	9.876125	45
16	819257	180743	943243	056757	123986	876014	44
17	819401	180599	943498	056502	124096	875904	43
18	819545	180455	943752	056248	124207	875793	42
19	819689	180311	944007	055993	124318	875682	41
20	9.819832	10.180168	9.944262	10.055738	10.124429	9.875571	40
21	819976	180024	944517	055483	124541	875459	39
22	820120	179880	944771	055229	124652	875348	38
23	820263	179737	945026	054974	124763	875237	37
24	820406	179594	945281	054719	124874	875126	36
25	9.820550	10.179450	9.945535	10.054465	10.124986	9.875014	35
26	820693	179307	945790	054210	125097	874903	34
27	820836	179164	946045	053955	125209	874791	33
28	820979	179021	946299	053701	125320	874680	32
29	821122	178878	946554	053446	125432	874568	31
30	9.821265	10.178735	9·946808	10.053192	10.125544	9.874456	30
31	821407	178593	947063	052937	125656	874344	29
32	821550	178450	947318	052682	125768	874232	28
33	821693	178307	947572	052428	125879	874121	27
34	821835	178165	947826	052174	125991	874009	26
35	9.821977	10.178023	9.948081	10.051919	10.126104	9.873896	25
36	822120	177880	948336	051664	126216	873784	24
37	822262	177738	948590	051410	126328	873672	23
38	822404	177596	948844	051156	126440	873560	22
39	822546	177454	949099	050901	126552	873448	21
40	9.822688	10.177312	9.949353	10.050647	10.126665	9.873335	20
41	822830	177170	949607	050393	126777	873223	19
42	822972	177028	949862	050138	126890	873110	18
43	823114	176886	950116	049884	127002	872998	17
44	823255	176745	950370	049630	127115	872885	16
45	9.823397	10.176603	9.950625	10.049375	10.127228	9.872772	15
46	823539	176461	950879	049121	127341	872659	14
47	823680	176320	951133	048867	127453	872547	13
48	823821	176179	951388	048612	127566	872434	12
49	823963	176037	951642	048358	127679	872321	11
50	9.824104	10.175896	9.951896	10.048104	10.127792	9.872208	10
51	824245	175755	952150	047850	127905	872095	9
52	824386	175614	952405	047595	128019	871981	8
53	824527	175473	952659	047341	128132	871868	7
54	824668	175332	952913	047087	128245	871755	6
55	9.824808	10.175192	9.953167	10.046833	10.128359	9.871641	5
56	824949	175051	953421	046579	128472	871528	4
57	825090	174910	953675	046325	128586	871414	3
58	825230	174770	953929	046071	128699	871301	2
59	825371	174629	954183	045817	128813	871187	1
60	825511	174489	954437	045563	128927	871073	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

48°

42°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.825511	10.174489	9.954437	10.045563	10.128927	9.871073	60
1	825651	174349	954691	045309	129040	870960	59
2	825791	174209	954945	045055	129154	870846	58
3	825931	174069	955200	044800	129268	870732	57
4	826071	173929	955454	044546	129382	870618	56
5	9.826211	10.173789	9.955707	10.044293	10.129496	9.870504	55
6	826351	173649	955961	044039	129610	870390	54
7	826491	173509	956215	043785	129724	870276	53
8	826631	173369	956469	043531	129839	870161	52
9	826770	173230	956723	043277	129953	870047	51
10	9.826910	10.173090	9.956977	10.043023	10.130067	9.869933	50
11	827049	172951	957231	042769	130182	869818	49
12	827189	172811	957485	042515	130296	869704	48
13	827328	172672	957739	042261	130411	869589	47
14	827467	172533	957993	042007	130526	869474	46
15	9.827606	10.172394	9.958246	10.041754	10.130640	9.869360	45
16	827745	172255	958500	041500	130755	869245	44
17	827884	172116	958754	041246	130870	869130	43
18	828023	171977	959008	040992	130985	869015	42
19	828162	171838	959262	040738	131100	868900	41
20	9.828301	10.171699	9.959516	10.040484	10.131215	9.868785	40
21	828439	171561	959769	040231	131330	868670	39
22	828578	171422	960023	039977	131445	868555	38
23	828716	171284	960277	039723	131560	868440	37
24	828855	171145	960531	039469	131676	868324	36
25	9.828993	10.171007	9.960784	10.039216	10.131791	9.868209	35
26	829131	170869	961038	038962	131907	868093	34
27	829269	170731	961291	038709	132022	867978	33
28	829407	170593	961545	038455	132138	867862	32
29	829545	170455	961799	038201	132253	867747	31
30	9.829683	10.170317	9.962052	10.037948	10.132369	9.867631	30
31	829821	170179	962306	037694	132485	867515	29
32	829959	170041	962560	037440	132601	867399	28
33	830097	169903	962813	037187	132717	867283	27
34	830234	169766	963067	036933	132833	867167	26
35	9.830372	10.169628	9.963320	10.036680	10.132949	9.867051	25
36	830509	169491	963574	036426	133065	866935	24
37	830646	169354	963827	036173	133181	866819	23
38	830784	169216	964081	035919	133297	866703	22
39	830921	169079	964335	035665	133414	866586	21
40	9.831058	10.168942	9.964588	10.035412	10.133530	9.866470	20
41	831195	168805	964842	035158	133647	866353	19
42	831332	168668	965095	034905	133763	866237	18
43	831469	168531	965349	034651	133880	866120	17
44	831606	168394	965602	034398	133996	866004	16
45	9.831742	10.168258	9.965855	10.034145	10.134113	9.865887	15
46	831879	168121	966109	033891	134230	865770	14
47	832015	167985	966362	033638	134347	865653	13
48	832152	167848	966616	033384	134464	865536	12
49	832288	167712	966869	033131	134581	865419	11
50	9.832425	10.167575	9.967123	10.032877	10.134698	9.865302	10
51	832561	167439	967376	032624	134815	865185	9
52	832697	167303	967629	032371	134932	865068	8
53	832833	167167	967883	032117	135050	864950	7
54	832969	167031	968136	031864	135167	864833	6
55	9.833105	10.166895	9.968389	10.031611	10.135284	9.864716	5
56	833241	166759	968643	031357	135402	864598	4
57	833377	166623	968896	031104	135519	864481	3
58	833512	166488	969149	030851	135637	864363	2
59	833648	166352	969403	030597	135755	864245	1
60	833783	166217	969656	030344	135873	864127	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

47°

43°

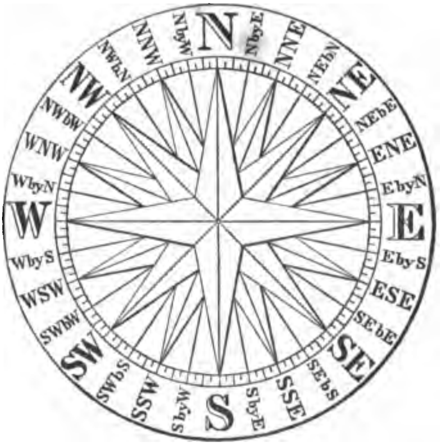
′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.833783	10.166217	9.969656	10.030344	10.135873	9.864127	60
1	833919	166081	969909	030091	135990	864010	59
2	834054	165946	970162	029838	136108	863892	58
3	834189	165811	970416	029584	136226	863774	57
4	834325	165675	970669	029331	136344	863656	56
5	9.834460	10.165540	9.970922	10.029078	10.136462	9.863538	55
6	834595	165405	971175	028825	136581	863419	54
7	834730	165270	971429	028571	136699	863301	53
8	834865	165135	971682	028318	136817	863183	52
9	834999	165001	971935	028065	136936	863064	51
10	9.835134	10.164866	9.972188	10.027812	10.137054	9.862946	50
11	835269	164731	972441	027559	137173	862827	49
12	835403	164597	972694	027306	137291	862709	48
13	835538	164462	972948	027052	137410	862590	47
14	835672	164328	973201	026799	137529	862471	46
15	9.835807	10.164193	9.973454	10.026546	10.137647	9.862353	45
16	835941	164059	973707	026293	137766	862234	44
17	836075	163925	973960	026040	137885	862115	43
18	836209	163791	974213	025787	138004	861996	42
19	836343	163657	974466	025534	138123	861877	41
20	9.836477	10.163523	9.974719	10.025281	10.138242	9.861758	40
21	836611	163389	974973	025027	138362	861638	39
22	836745	163255	975226	024774	138481	861519	38
23	836878	163122	975479	024521	138600	861400	37
24	837012	162988	975732	024268	138720	861280	36
25	9.837146	10.162854	9.975985	10.024015	10.138839	9.861161	35
26	837279	162721	976238	023762	138959	861041	34
27	837412	162588	976491	023509	139078	860922	33
28	837546	162454	976744	023256	139198	860802	32
29	837679	162321	976997	023003	139318	860682	31
30	9.837812	10.162188	9.977250	10.022750	10.139438	9.860562	30
31	837945	162055	977503	022497	139558	860442	29
32	838078	161922	977756	022244	139678	860322	28
33	838211	161789	978009	021991	139798	860202	27
34	838344	161656	978262	021738	139918	860082	26
35	9.838477	10.161523	9.978515	10.021485	10.140038	9.859962	25
36	838610	161390	978768	021232	140158	859842	24
37	838742	161258	979021	020979	140279	859721	23
38	838875	161125	979274	020726	140399	859601	22
39	839007	160993	979527	020473	140520	859480	21
40	9.839140	10.160860	9.979780	10.020220	10.140640	9.859360	20
41	839272	160728	980033	019967	140761	859239	19
42	839404	160596	980286	019714	140881	859119	18
43	839536	160464	980538	019462	141002	858998	17
44	839668	160332	980791	019209	141123	858877	16
45	9.839800	10.160200	9.981044	10.018956	10.141244	9.858756	15
46	839932	160068	981297	018703	141365	858635	14
47	840064	159936	981550	018450	141486	858514	13
48	840196	159804	981803	018197	141607	858393	12
49	840328	159672	982056	017944	141728	858272	11
50	9.840459	10.159541	9.982309	10.017691	10.141849	9.858151	10
51	840591	159409	982562	017438	141971	858029	9
52	840722	159278	982814	017186	142092	857908	8
53	840854	159146	983067	016933	142214	857786	7
54	840985	159015	983320	016680	142335	857665	6
55	9.841116	10.158884	9.983573	10.016427	10.142457	9.857543	5
56	841247	158753	983826	016174	142578	857422	4
57	841378	158622	984079	015921	142700	857300	3
58	841509	158491	984331	015669	142822	857178	2
59	841640	158360	984584	015416	142944	857056	1
60	841771	158229	984837	015163	143066	856934	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

46°

44°

′	Sine.	Cosec.	Tang.	Cotang.	Secant.	Cosine.	′
0	9.841771	10.158229	9.984837	10.015163	10.143066	9.856934	60
1	841902	158098	985090	014910	143188	856812	59
2	842033	157967	985343	014657	143310	856690	58
3	842163	157837	985596	014404	143432	856568	57
4	842294	157706	985848	014152	143554	856446	56
5	9.842424	10.157576	9.986101	10.013899	10.143677	9.856323	55
6	842555	157445	986354	013646	143799	856201	54
7	842685	157315	986607	013393	143922	856078	53
8	842815	157185	986860	013140	144044	855956	52
9	842946	157054	987112	012888	144167	855833	51
10	9.843076	10.156924	9.987365	10.012635	10.144289	9.855711	50
11	843206	156794	987618	012382	144412	855588	49
12	843336	156664	987871	012129	144535	855465	48
13	843466	156534	988123	011877	144658	855342	47
14	843595	156405	988376	011624	144781	855219	46
15	9.843725	10.156275	9.988629	10.011371	10.144904	9.855096	45
16	843855	156145	988882	011118	145027	854973	44
17	843984	156016	989134	010866	145150	854850	43
18	844114	155886	989387	010613	145273	854727	42
19	844243	155757	989640	010360	145397	854603	41
20	9.844372	10.155628	9.989893	10.010107	10.145520	9.854480	40
21	844502	155498	990145	009855	145644	854356	39
22	844631	155369	990398	009602	145767	854233	38
23	844760	155240	990651	009349	145891	854109	37
24	844889	155111	990903	009097	146014	853986	36
25	9.845018	10.154982	9.991156	10.008844	10.146138	9.853862	35
26	845147	154853	991409	008591	146262	853738	34
27	845276	154724	991662	008338	146386	853614	33
28	845405	154595	991914	008086	146510	853490	32
29	845533	154467	992167	007833	146634	853366	31
30	9.845662	10.154338	9.992420	10.007580	10.146758	9.853242	30
31	845790	154210	992672	007328	146882	853118	29
32	845919	154081	992925	007075	147006	852994	28
33	846047	153953	993178	006822	147131	852869	27
34	846175	153825	993430	006570	147255	852745	26
35	9.846304	10.153696	9.993683	10.006317	10.147380	9.852620	25
36	846432	153568	993936	006064	147504	852496	24
37	846560	153440	994189	005811	147629	852371	23
38	846688	153312	994441	005559	147753	852247	22
39	846816	153184	994694	005306	147878	852122	21
40	9.846944	10.153056	9.994947	10.005053	10.148003	9.851997	20
41	847071	152929	995199	004801	148128	851872	19
42	847199	152801	995452	004548	148253	851747	18
43	847327	152673	995705	004295	148378	851622	17
44	847454	152546	995957	004043	148503	851497	16
45	9.847582	10.152418	9.996210	10.003790	10.148628	9.851372	15
46	847709	152291	996463	003537	148754	851246	14
47	847836	152164	996715	003285	148879	851121	13
48	847964	152036	996968	003032	149004	850996	12
49	848091	151909	997221	002779	149130	850870	11
50	9.848218	10.151782	9.997473	10.002527	10.149255	9.850745	10
51	848345	151655	997726	002274	149381	850619	9
52	848472	151528	997979	002021	149507	850493	8
53	848599	151401	998231	001769	149632	850368	7
54	848726	151274	998484	001516	149758	850242	6
55	9.848852	10.151148	9.998737	10.001263	10.149884	9.850116	5
56	848979	151021	998989	001011	150010	849990	4
57	849106	150894	999242	000758	150136	849864	3
58	849232	150768	999495	000505	150262	849738	2
59	849359	150641	999747	000253	150389	849611	1
60	849485	150515	10.00000	000000	150515	849485	0
′	Cosine.	Secant.	Cotang.	Tang.	Cosec.	Sine.	′

45°



POINTS AND QUARTER-POINTS REDUCED TO DEGREES, ETC.

North.		P. Q.	D. M. S.	P. Q.	South.	
		0 1	2 48 45	0 1		
		0 2	5 37 30	0 2		
		0 3	8 26 15	0 3		
N. by E.	N. by W.	1 0	11 15 0	1 0	S. by E.	S. by W.
		1 1	14 3 45	1 1		
		1 2	16 52 30	1 2		
		1 3	19 41 15	1 3		
N.N.E.	N.N.W.	2 0	22 30 0	2 0	S.S.E.	S.S.W.
		2 1	25 18 45	2 1		
		2 2	28 7 30	2 2		
		2 3	30 56 15	2 3		
N.E. by N.	N.W. by N.	3 0	33 45 0	3 0	S.E. by S.	S.W. by S.
		3 1	36 33 45	3 1		
		3 2	39 22 30	3 2		
		3 3	42 11 15	3 3		
N.E.	N.W.	4 0	45 0 0	4 0	S.E.	S.W.
		4 1	47 48 45	4 1		
		4 2	50 37 30	4 2		
		4 3	53 26 15	4 3		
N.E. by E.	N.W. by W.	5 0	56 15 0	5 0	S.E. by E.	S.W. by W.
		5 1	59 3 45	5 1		
		5 2	61 52 30	5 2		
		5 3	64 41 15	5 3		
E.N.E.	W.N.W.	6 0	67 30 0	6 0	E.S.E.	W.S.W.
		6 1	70 18 45	6 1		
		6 2	73 7 30	6 2		
		6 3	75 56 15	6 3		
E. by N.	W. by N.	7 0	78 45 0	7 0	E. by S.	W. by S.
		7 1	81 33 45	7 1		
		7 2	84 22 30	7 2		
		7 3	87 11 15	7 3		
East.	West.	8 0	90 0 0	8 0	East.	West.

TABLE VI.

Dip of the Sea Horizon.

H is the height of the eye in feet.

H.	Dip.	H.	Dip.
1	0′ 59″	38	6′ 04″
2	1 24	41	6 18
3	1 42	44	6 32
4	1 58	47	6 45
5	2 12	50	6 58
6	2 25	53	7 10
7	2 36	56	7 22
8	2 47	59	7 34
9	2 57	62	7 45
10	3 07	65	7 56
11	3 16	68	8 07
12	3 25	71	8 18
13	3 33	74	8 28
14	3 41	77	8 38
15	3 49	80	8 48
16	3 56	83	8 58
17	4 04	86	9 08
18	4 11	89	9 17
19	4 17	92	9 26
20	4 24	95	9 36
21	4 31	100	9 51
22	4 37	110	10 19
23	4 43	120	10 47
24	4 49	130	11 14
25	4 55	140	11 39
26	5 01	150	12 04
27	5 07	200	13 56
28	5 13	300	17 04
30	5 24	400	19 42
32	5 34	500	22 01
35	5 49	1000	31 09

TABLE VIII.

Correction, in Levelling, for the Curvature of the Earth.

Dist.	Cor.	Dist.	Cor.
Chains.	Inches.	Chains.	Inches.
1	0·001	32	1·281
2	0·005	34	1·45
3	0·011	36	1·62
4	0·020	38	1·81
5	0·031	40	2·00
6	0·045	45	2·53
7	0·061	50	3·13
8	0·080	55	3·78
9	0·101	60	4·50
10	0·125	65	5·29
11	0·151	70	6·13
12	0·180	75	7·04
13	0·211	80	8·01
14	0·245	85	9·04
15	0·282	90	10·13
16	0·320	95	11·29
17	0·362	100	12·51
18	0·405	Imp. Miles.	Feet.
19	0·452		
20	0·500	1	0·67
21	0·552	2	2·67
22	0·606	3	6·01
23	0·662	4	10·7
24	0·721	5	16·7
25	0·782	6	24·0
26	0·846	7	32·7
27	0·912	8	42·7
28	0·981	9	54·1
29	1·052	10	66·7
30	1·126	11	80·7
31	1·202	12	96·1

TABLE VII.

Dip of the Sea at different Distances.

Dist. in Naut. Miles.	Height of the eye in feet.					
	5	10	15	20	25	30
0¼	11′	22′	34′	45′	56′	68′
0½	6	11	17	22	28	34
0¾	4	8	12	15	19	23
1	4	6	9	12	15	17
1¼	3	5	7	9	12	14
1½	3	4	6	8	10	12
2	2	3	5	6	8	10
2½	2	3	5	6	7	8
3	2	3	4	5	6	7
3½	2	3	4	5	6	6
4	2	3	4	4	5	6
5	2	3	4	4	5	5
6	2	3	4	4	5	5

TABLE IX.

Deduction from each Chain, to reduce oblique to horizontal Measurement.

Degrees.	Links.	Degrees.	Links.
1	0·02	14	2·97
2	0·06	15	3·41
3	0·14	16	3·87
4	0·24	17	4·37
5	0·38	18	4·89
6	0·55	19	5·45
7	0·75	20	6·03
8	0·98	21	6·64
9	1·23	22	7·28
10	1·52	23	7·95
11	1·84	24	8·65
12	2·19	25	9·37
13	2·56	26	10·12

NOTE.—In Tables VI and VII allowance is made for refraction, but not in Table VIII.

FINIS.

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